MSC2020: 49N79, 49N70, 91A24

© ***A. A. Azamov, B. T. Samatov, N. T. Umaraliyeva***

Π-STRATEGY ALGORITHM IN DISCRETE PURSUIT GAMES

This paper is devoted to the study of one well-known problem of B. N. Pshenichnyi, namely the problem of simple group pursuit, when players make step-by-step movements. The paper considers two separate cases. In the first case, a discrete pursuit game is solved, when only one pursuer and one evader participate in the game. To solve this problem, an algorithm for applying the Π-strategy is given. According to the proposed method, the players first approach each other and eventually coincide exactly. In the second case, the proposed solution method is extended to the game of group pursuit. The obtained results are verified using animation models created in the Visual $C\#$ programming language using ScottPlot.WinForms technology.

Keywords: discrete game, pursuer, evader, strategy, pursuit, guaranteed capture step.

DOI: 10.35634/vm260101

Introduction

As is known, the Theory of Dynamic Games is mainly divided into two interrelated classes: 1) Differential Games, in which the equations of players' motion are described by differential equations and the players' strategies continuously depend on incoming information; 2) Discrete Games, in which the equations of players' motion are described by step-by-step recurrent equations and the players' strategies in this case must predetermine the choice of moves at each step.

At first, the attention of specialists was focused mainly on differential games, and in this direction, various approaches to solving game problems were proposed. The founder of this theory, R. Isaacs [1], proposed solution methods based on variational principles, where the main role is played by the Hamilton–Jacobi–Bellman equations. Later, interesting results were obtained in this direction, for example in the works [2–4]. In the works [5, 6], differential games are considered as special classes of control problems, where differential games are considered from two points of view, from the point of view of the pursuer, as a pursuit problem, and from the point of view of the evader, as an evasion problem.

If we move from differential games to discrete games, there are also quite a lot of interesting works in this direction today, however, compared to differential games, there are still quite a few discrete problems that require further research. More detailed information on discrete games can be found in works [7–11].

In works [12–18], dynamic processes are considered from a more general point of view, since differential and discrete processes have much in common. In these works, continuous and discrete time intervals are considered from a single point of view as a set of time scales. Such an approach to solving general dynamic systems was proposed in works [12–14] and this remarkable way of generalizing continuous and discrete processes found its application in the theory of dynamic games [15–18].

This paper is devoted to the study of one well-known problem of B. N. Pshenichnyi [19] — the problem of simple pursuit with step-by-step movement of players. In solving this problem, the key role is played by the strategy of parallel pursuit (or, in short, the Π-strategy [20]), which led to the creation of one of the widely used methods for solving group pursuit problems — the method of resolving functions [21–30]. The paper considers two separate cases. In the first case, a discrete pursuit game is solved, when only one pursuer and one evader participate in the game. To solve this problem, an algorithm for applying the Π-strategy is given. According to the proposed method, the players first approach each other and eventually coincide exactly. In the second case, the proposed solution method is extended to the game of group pursuit. The obtained results are verified using animation models created in the Visual $C\#$ programming language using ScottPlot.WinForms technology [31,32].

§ 1. Statement of the problem

In space $\mathbb{R}^d$, two objects perform stepwise movements. The goal of the first object P (the pursuer) is to reach or coincide with the second object E (the evader) in the final step. The purpose of the second object is the opposite. Let their movements be described by the following recurrent equations

$$P\colon x_n = x_{n-1} + u, \tag{1.1}$$

$$E\colon y_n = y_{n-1} + v, \tag{1.2}$$

respectively. Here: x_n is the location position of P in space $\mathbb{R}^d$ at step n, $n = 1, 2, \ldots$; y_n is the location position of E in space $\mathbb{R}^d$ at step n, $n = 1, 2, \ldots$; x_0 is the initial position of P; y_0 is the initial position of E; u is a control parameter of P, which at each step takes only one arbitrary vector value from S_α, where S_α is a ball of radius α with the center at the origin of the coordinate space $\mathbb{R}^d$; v is a control parameter of E, which at each step takes only one arbitrary vector value from S_β, where $S_\beta \subset \mathbb{R}^d$ is also a ball of radius β.

Let an arbitrary sequence of controls of players P and E be chosen in the form of u_n and v_n, respectively. Then, substituting these controls in recurrent equations (1.1) and (1.2), we have solutions to these equations in the form of the following sequences:

$$P\colon x_n = x_0 + \sum_{i=1}^{n} u_i, \tag{1.3}$$

$$E\colon y_n = y_0 + \sum_{i=1}^{n} v_i, \tag{1.4}$$

where $n = 1, 2, \ldots$. Sequences (1.3) and (1.4) will be called traces of players P and E in space $\mathbb{R}^d$, respectively. Then, the goal of the pursuer P is to realize the equality $x_N = y_N$ for some finite step N, $N = 1, 2, \ldots$ (Discrete Pursuit Game), and the goal of the evader is to avoid this equality, that is, to realize the inequality $x_n \neq y_n$ for all $n = 1, 2, \ldots$ (Discrete Evasion Game). This is the preliminary statement of the Pursuit–Evasion problems for the considered game.

Similarly to the continuous problem and here, to simplify the solution of the discrete pursuit problem, we introduce a new variable $z_n = x_n - y_n$ for all $n = 1, 2, \ldots$. Then, instead of recurrent equations (1.1)–(1.2) and their solutions (1.3)–(1.4), we obtain equation

$$z_n = z_{n-1} + u - v, \tag{1.5}$$

and, for given sequences u_n and v_n, we have the solution

$$z_n = z_0 + \sum_{i=1}^{n} (u_i - v_i), \tag{1.6}$$

where $z_0 = x_0 - y_0$. In this case, the pursuer P strives to implement the equality $z_n = 0$ up to a certain finite number of the step N, where $n \leqslant N$ (Discrete Pursuit Problem), and the evader E strives to implement the inequality $z_n \neq 0$ for all $n = 1, 2, \ldots$ or avoid this meeting for as long as possible (Discrete Evasion Problem). This is a preliminary formulation of Discrete Pursuit and Evasion Problems.

§ 2. Solving Discrete Pursuit Problem

2.1. Pursuer Strategy

To solve the Discrete Pursuit Problem (DPP) (1.1)–(1.2), we assume that the pursuer is allowed to know the initial states x_0, y_0, the constants α, β and the value of the sequences v_n and z_{n-1} on each current step $n = 1, 2, \ldots$.

Definition 2.1. We will call the sequence

$$\boldsymbol{u}_n = \begin{cases} \boldsymbol{u}_n^A, & \text{if } |z_{n-1}| > \mu(v_n), \\ \boldsymbol{u}_n^F, & \text{if } |z_{n-1}| \leqslant \mu(v_n), \end{cases} \tag{2.1}$$

Π-*strategy* in the DPP (1.1)–(1.2) or (1.5), where

$$\boldsymbol{u}_n^A = v_n - \mu(v_n)\xi_0, \tag{2.2}$$

$$\boldsymbol{u}_n^F = v_n - z_{n-1}, \tag{2.3}$$

$$\mu(v_n) = \langle v_n, \xi_0 \rangle + \sqrt{\langle v_n, \xi_0 \rangle^2 + \alpha^2 - |v_n|^2}, \tag{2.4}$$

$\xi_0 = z_0/|z_0|$, $\langle v_n, \xi_0 \rangle$ means the inner product of the vectors v_n and ξ_0 in $\mathbb{R}^d$, $n = 1, 2, \ldots$. Further, we will call sequence $\boldsymbol{u}_n^A$ *the approach strategy*, and sequence $\boldsymbol{u}_n^F$ *the final* or *finishing strategy*.

From the definition of the Π-strategy (2.1) it is obvious that the pursuer's strategy at each step n depends on the sequences v_n and z_{n-1}.

Lemma 2.1. *If $\alpha \geqslant \beta$, then for each pair $(v_n, z_{n-1}) \in S_\beta \times \mathbb{R}^d$:*

(a) *the sequence $\mathbf{u}_n$ is well defined;*

(b) *the inequalities $\alpha - \beta \leqslant \mu(v_n) \leqslant \alpha + \beta$ hold for any $v_n \in S_\beta$.*

The proof of the lemma follows directly from forms (2.1)–(2.3).

2.2. Pursuit Algorithm

Let the condition $\alpha > \beta$ be satisfied and the pursuer implements strategy (2.1). Let us consider the actions of the pursuer at each step $n \geqslant 1$.

Step 1. Let at step 1 the evader choose some vector $v_1 \in S_\beta$ and for this vector and the initial state z_0 the inequality $|z_0| > \mu(v_1)$ holds. From Definition 2.1 it follows that the pursuer in this case chooses the vector $\boldsymbol{u}_1^A = v_1 - \mu(v_1)\xi_0$ (see (2.2)). Consequently, from (1.6) we have the equality

$$z_1 = z_0 + \boldsymbol{u}_1^A - v_1 = z_0 - \mu(v_1)\xi_0. \tag{2.5}$$

Note that vectors z_1 and z_0 are collinear, or in other words, parallel to each other.

Let us show that for an arbitrary choice of vector v_1 from the ball S_β, the inequality $z_1 \neq 0$ is satisfied. Let us assume the opposite, i. e., there is some control vector v_1^* for which $z_1 = 0$. Then, from (2.5), we get

$$0 = |z_1| = |z_0| - \mu(v_1^*),$$

which contradicts the assumption.

Now let the inequality $|z_0| \leqslant \mu(v_1)$ be satisfied at step 1. Then, from (1.6) and (2.3), we have

$$z_1 = z_0 + \boldsymbol{u}_1^F - v_1 = 0,$$

i. e., the pursuit ends in step 1.

Thus, if in step 1 the inequality $|z_0| > \mu(v_1)$ is satisfied, then the pursuer continues to apply strategy (2.1) in step 2.

Step 2. Let the inequality $|z_1| > \mu(v_2)$ also be satisfied in step $n = 2$. Then, from (1.6), (2.2) and (2.5), we have

$$z_2 = z_0 + \boldsymbol{u}_1^A - v_1 + \boldsymbol{u}_2^A - v_2 = z_0 - \mu(v_1)\xi_0 - \mu(v_2)\xi_0 = z_1 - \mu(v_2)\xi_0.$$

Note that here too the vectors z_2, z_1 and z_0 are collinear with each other. Then, similarly to step 1, here we can also prove that if the inequality $|z_1| > \mu(v_2)$ holds, then for any vector $v_2 \in S_\beta$ the inequality $z_2 \neq 0$ holds, i. e., it is impossible to catch the evader at step 2.

If in the second step the inequality $|z_1| \leqslant \mu(v_2)$ is satisfied, then, from (1.6), (2.3) and (2.5), it follows that

$$z_2 = z_0 + \boldsymbol{u}_1^A - v_1 + \boldsymbol{u}_2^F - v_2 = z_1 + \boldsymbol{u}_2^F - v_2 = 0,$$

i. e., the pursuit ends in step 2.

Inductively continuing this process, one can reach some step n.

Step *n*. Let the inequality $|z_{n-1}| > \mu(v_n)$ also be satisfied in step $n > 2$. Then, from (1.6) and (2.2), we obtain

$$z_n = z_0 + \sum_{i=1}^{n}(\boldsymbol{u}_i^A - v_i) = z_0\biggl(1 - \frac{1}{|z_0|}\sum_{i=1}^{n}\mu(v_i)\biggr), \tag{2.6}$$

i. e., vectors z_i, $i = 1, 2, \dots, n$, remain parallel to the initial vector z_0. Based on this property, similarly to step 1, here it can be proved that in this case it is also impossible to complete the pursuit, i. e., $z_n \neq 0$.

In the case when at steps $i = 1, 2, \dots, n-1$ the inequalities $|z_{i-1}| > \mu(v_i)$ are satisfied and, at the last step, $|z_{n-1}| \leqslant \mu(v_n)$, then, from (1.5) and (2.3), we have

$$z_n = z_{n-1} + \boldsymbol{u}_n^F - v_n = 0,$$

i. e., the pursuit ends in step n.

Now we will prove the admissibility of strategy (2.1), i. e., the validity of the inequality $|\boldsymbol{u}_n| \leqslant \alpha$ for any $n \geqslant 1$. For the case $|z_{n-1}| > \mu(v_n)$, from the definition of sequence (2.2) we have

$$|\boldsymbol{u}_n^A|^2 = |v_n|^2 - 2\mu(v_n)\langle v_n, \xi_0\rangle + \mu^2(v_n) = |v_n|^2 + \mu(v_n)\bigl(\mu(v_n) - 2\langle v_n, \xi_0\rangle\bigr).$$

Hence, from (2.4), we get

$$|\boldsymbol{u}_n^A|^2 = |v_n|^2 + \mu(v_n)\bigl(\sqrt{\langle v_n, \xi_0\rangle^2 + \alpha^2 - |v_n|^2} - \langle v_n, \xi_0\rangle\bigr) =$$
$$= |v_n|^2 + \bigl(\sqrt{\langle v_n, \xi_0\rangle^2 + \alpha^2 - |v_n|^2} + \langle v_n, \xi_0\rangle\bigr)\bigl(\sqrt{\langle v_n, \xi_0\rangle^2 + \alpha^2 - |v_n|^2} - \langle v_n, \xi_0\rangle\bigr) = \alpha^2.$$

Now, for the case $|z_{n-1}| \leqslant \mu(v_n)$, we have

$$\langle v_n, \xi_0\rangle - \sqrt{\langle v_n, \xi_0\rangle^2 + \alpha^2 - |v_n|^2} \leqslant |z_{n-1}| \leqslant \mu(v_n) \Rightarrow$$
$$\Rightarrow \bigl||z_{n-1}| - \langle v_n, \xi_0\rangle\bigr| \leqslant \sqrt{\langle v_n, \xi_0\rangle^2 + \alpha^2 - |v_n|^2}.$$

We square both sides of the last inequality and obtain

$$|z_{n-1}|^2 - 2|z_{n-1}|\langle v_n, \xi_0\rangle + |v_n|^2 \leqslant \alpha^2.$$

From equality (2.6), it follows that vectors z_{n-1} and z_0 have the same directions and therefore $z_{n-1} = |z_{n-1}|\xi_0$. Hence, we have $|v_n - z_{n-1}| \leqslant \alpha$. Q. E. D. □

From the above we formulate the following theorem.

Theorem 2.1. *Let $\alpha > \beta$ and the pursuer implements the Π-strategy* (2.1)*.*

(a) *If for steps $i = 1, 2, \dots, n$ the inequalities $|z_{i-1}| > \mu(v_i)$ are satisfied, then, for all these steps $i = 1, 2, \dots, n$, we have $z_i \neq 0$.*

(b) *If inequality $|z_n| \leqslant \mu(v_{n+1})$ is satisfied in step n, then in step $n+1$ the evader is captured, i. e., $z_{n+1} = 0$.*

2.3. Guaranteed pursuit step

First, let us prove the following important lemma.

Lemma 2.2. *If at some step n the inequalities $0 < |z_n| \leqslant \alpha - \beta$ are satisfied, then the pursuer, using the final strategy (2.3), completes the pursuit at step $n+1$.*

P r o o f. If the conditions of the lemma are met, then, for any vector $v_{n+1} \in S_\beta$,

$$\alpha \geqslant \beta + |z_n| \geqslant |v_{n+1}| + |z_n| \geqslant |v_{n+1} - z_n| \Rightarrow |z_n| \leqslant \mu(v_{n+1}).$$

Consequently, by virtue of Theorem 2.1, (b), the game ends at step $n+1$. □

Theorem 2.2. *Let $\alpha > \beta$ be satisfied in the discrete game (1.1)–(1.2) (or (1.5)). Then the pursuer, implementing the Π-strategy (2.1), completes the pursuit no later than in step $N = \left[\frac{|z_0|}{\alpha-\beta}\right] + 1$, where the sign $[\cdot]$ denotes the integer part of the number inside this sign.*

P r o o f. Let the evader choose an arbitrary sequence of vectors $v_k \in S_\beta$, where $k = 1, 2, \ldots$. By virtue of Theorem 2.1, the pursuer can implement strategy (2.1) up to some step n. Then, from (2.6) and Lemma 2.1, we have the estimate

$$|z_n| = |z_0| - \sum_{i=1}^{n} \mu(v_i) \leqslant |z_0| - n(\alpha - \beta).$$

From these relations it follows that there exists a minimal step n^* for which the inequality $|z_0| - n^*(\alpha - \beta) \leqslant \alpha - \beta$ or $\frac{|z_0|}{\alpha-\beta} - 1 \leqslant n^*$ holds. Obviously, $n^* = \left[\frac{|z_0|}{\alpha-\beta}\right]$. Then, by Lemma 2.2, we obtain that the pursuit ends no later than step N. □

§ 3. Group Discrete Pursuit Problem

3.1. Problem formulation

In this section, we consider the Group Discrete Pursuit Problem (GDPP) with simple movement of players, i. e., a discrete analogue of the Pshenichnyi Problem [19]. Let in space $\mathbb{R}^d$ objects P^1, P^2, …, P^m (the pursuers) step-by-step pursue object E (the evader), which also moves step-by-step. The movements of these objects are described by the following recurrence equations

$$P^i\colon x_n^i = x_{n-1}^i + u^i, \quad i \in I_m = \{1, 2, 3, \ldots, m\}, \tag{3.1}$$

$$E\colon y_n = y_{n-1} + v; \tag{3.2}$$

everywhere below $n = 1, 2, \ldots$; u^i are the control parameters of the pursuers, which at each step n are chosen as constant vectors u_n^i, satisfying the so-called geometric constraints of the form

$$|u_n^i| \leqslant 1, \quad i \in I_m; \tag{3.3}$$

v is the control parameter of the evader, which at each step n is chosen as a constant vector v_n satisfying the constraint

$$|v_n| \leqslant 1, \tag{3.4}$$

i. e., the pursuers and the evader have the same capabilities; x_0^i, $i \in I_m$, and y_0 are the initial states of the pursuers and the evader, respectively. The goal of the pursuers is to achieve the equality $x_n^j = y_n$ for at least one pursuer $j \in I_m$ at some finite step n. The goal of the evader is the opposite, i. e., to achieve the inequality $x_n^i \neq y_n$ for all pursuers $i \in I_m$ at any step n.

Similar to Section 1, here we simplify the solution of the discrete pursuit problem by introducing new variables: $z_n^i = x_n^i - y_n$, $i \in I_m$, and $z_0^i = x_0^i - y_0$. Then, from (3.1)–(3.2), we obtain the recurrence equations

$$z_n^i = z_{n-1}^i + u_n^i - v_n, \quad i \in I_m. \tag{3.5}$$

The solutions to equations (3.5) have the forms

$$z_n^i = z_0^i + \sum_{k=1}^{n} (u_k^i - v_k), \quad i \in I_m. \tag{3.6}$$

The solution of the discrete pursuit problem is the realization of the equality $z_n^j = 0$ for some $j \in I_m$ at the final step of n, and the solution of the evasion problem is the realization of the inequality $z_n^i \neq 0$ for all $i \in I_m$ at any step n.

Definition 3.1. The sequence

$$\boldsymbol{u}_n^i = \begin{cases} \boldsymbol{u}_n^{A_i}, & \text{if } |z_{n-1}^i| > \mu_i(v_n), \\ \boldsymbol{u}_n^{F_i}, & \text{if } |z_{n-1}^i| \leqslant \mu_i(v_n), \end{cases} \tag{3.7}$$

will be called *the* Π_i*-strategy* of the i-pursuer in the GDPP (3.1)–(3.2) or (3.5), where

$$\boldsymbol{u}_n^{A_i} = v_n - \mu_i(v_n)\xi_0^i, \tag{3.8}$$

$$\boldsymbol{u}_n^{F_i} = v_n - z_{n-1}^i, \tag{3.9}$$

$$\mu_i(v_n) = \langle v_n, \xi_0^i \rangle + \sqrt{\langle v_n, \xi_0^i \rangle^2 + 1 - |v_n|^2}, \quad n = 1, 2, \ldots, \tag{3.10}$$

$\xi_0^i = z_0^i / |z_0^i|$, $i \in I_m$.

3.2. Solution for the Pursuit Problem

Theorem 3.1. *Let in the game* (3.1)–(3.5) *the pursuers implement* Π_i*-strategies* (3.7).

(a) *If for steps* $k = 1, 2, \ldots, n$ *the inequalities* $|z_{k-1}^i| > \mu_i(v_k)$, $i \in I_m$, *are satisfied, then for any step* $k = 1, 2, \ldots, n$ *we have* $z_k^i \neq 0$ *for all* $i \in I_m$.

(b) *If at step* n *the inequality* $|z_n^j| \leqslant \mu_j(v_{n+1})$ *holds for some* $j \in I_m$, *then at step* $n+1$ *the pursuer* P^j *completes the pursuit, i. e.,* $z_{n+1}^j = 0$.

The proof of Theorem 3.1 is similar to the proof of Theorem 2.1.

Theorem 3.2 (Discrete analogue of Pshenichyi's Theorem [19]). *If in the game* (3.1)–(3.5) *for the initial states* z_0^i, $i \in I_m$, *the condition*

$$0 \in \operatorname{int} \operatorname{conv} \{\xi_0^1, \xi_0^2, \ldots, \xi_0^m\} \tag{3.11}$$

is satisfied, then with the help of Π_i*-strategies* (3.7) *the pursuers complete the pursuit no later than step* $N = \left[\sum_{i=1}^{m} |z_0^i| / \gamma \right] + 1$, *where* $\gamma = 2\delta/(1 + 2\delta)$ *and*

$$\delta = \min_{|p|=1} \max_{i \in I_m} \langle p, \xi_0^i \rangle > 0. \tag{3.12}$$

P r o o f. From condition (3.11), it follows the existence of a positive number δ. Thus, $\max\limits_{i\in I_m}\langle p,\xi_0^i\rangle \geqslant \delta$ for every p, $|p|=1$.

It is easy to verify that the condition $|v_n|\leqslant 1$ implies the estimate

$$\mu_i(v_n)\geqslant \max\bigl\{1-|v_n|, 2\langle v_n,\xi_0^i\rangle\bigr\} \tag{3.13}$$

for (3.10). Indeed, if $|v_n|=1$, then $\mu_i(v_n)=\max\{0, 2\langle v_n,\xi_0^i\rangle\}$ and equality holds in (3.13). If $|v_n|<1$, then we obtain the inequalities $\mu_i(v_n)\geqslant 1-|v_n|$ and $\mu_i(v_n)>2\langle v_n,\xi_0^i\rangle$ or (3.13). Then, assuming $p_n=v_n/|v_n|$ for $v_n\neq 0$, we obtain

$$\max_{i\in I_m}\mu_i(v_n)\geqslant \max\bigl\{1-|v_n|, 2|v_n|\max_{i\in I_m}\langle p_n,\xi_0^i\rangle\bigr\}\geqslant \max\bigl\{1-|v_n|, 2|v_n|\delta\bigr\}\geqslant\gamma \tag{3.14}$$

for all $n=1,2,\ldots$. The last inequality follows from the fact that a continuous function $f(\nu)=$ $=\max\{1-\nu, 2\nu\delta\}$ on an interval $[0,1]$ reaches its minimum value at point $\nu_0=1/(1+2\delta)$, i. e., $\min\limits_{\nu\in[0,1]} f(\nu)=\gamma$, where $\nu=|v_n|$.

Now let's estimate the distance between the pursuers and the evader at each step n. By Theorem 3.1, (a), let the pursuers apply strategy (3.8) up to some step n. Then from (3.6) we have

$$z_n^i = z_0^i-\sum_{k=1}^{n}\mu_i(v_k)\xi_0^i, \quad i\in I_m.$$

Hence, we get

$$|z_n^i| = |z_0^i|-\sum_{k=1}^{n}\mu_i(v_k), \quad i\in I_m.$$

From here and from inequality (3.14), we obtain the estimate

$$\sum_{i=1}^{m}|z_n^i| = \sum_{i=1}^{m}|z_0^i|-\sum_{i=1}^{m}\sum_{k=1}^{n}\mu_i(v_k)\leqslant \sum_{i=1}^{m}|z_0^i|-\sum_{k=1}^{n}\max_{i\in I_m}\mu_i(v_k)\leqslant \sum_{i=1}^{m}|z_0^i|-n\gamma.$$

Similar to the proof of Theorem 2.2, here in step $n^*=\left[\sum\limits_{i=1}^{m}|z_0^i|/\gamma\right]$ the inequality

$$\sum_{i=1}^{m}|z_{n^*}^i|\leqslant \sum_{i=1}^{m}|z_0^i|-n^*\gamma\leqslant\gamma\leqslant\max_{i\in I_m}\mu_i(v_{n^*+1}).$$

is satisfied. It follows that there is at least one j-pursuer P^j, for which the relations $|z_{n^*}^j|\leqslant\mu_j(v_{n^*+1})$ holds. Then from Theorem 3.1, it follows that the pursuit is completed using strategy (3.9) by the j-pursuer at step $N=\left[\sum\limits_{i=1}^{m}|z_0^i|/\gamma\right]+1$. □

§ 4. Program implementations and examples

4.1. Discrete Pursuit Problem: algorithm and implementation

In this subsection, a software algorithm for solving the pursuit problem in the plane is proposed for the case of one pursuer P and one evader E.

Step 1. The algorithm starts with the following input data: the coordinates of the initial states of the players $E\colon y_0 := (y_{01}, y_{02})$, $P\colon x_0 := (x_{01}, x_{02})$ and the upper limits on the speed values α, β.

Step 2. The following are computed: $z_{01} := x_{01} - y_{01}$, $z_{02} := x_{02} - y_{02}$, $|z_0| := \sqrt{z_{01}^2 + z_{02}^2}$.

Step 3. **If** $(\alpha > \beta \wedge \alpha > 0 \wedge \beta \geqslant 0 \wedge |z_0| \neq 0)$
then compute the Guaranteed Pursuit Step (GPS) $N = \left[\frac{|z_0|}{\alpha-\beta}\right] + 1$;
set $n := 1$;
else return to Step 1.

Step 4. Enter evader vector $v_n := (v_{n1}, v_{n2})$.

Step 5. Compute $|v_n| := \sqrt{v_{n1}^2 + v_{n2}^2}$.

Step 6. **If** $|v_n| \leqslant \beta$
then compute $\mu(v_n)$ (2.4);
compute the coordinates for moving to the next step $E\colon y_n$ (1.2);
else return to Step 4.

Step 7. **If** $|z_{n-1}| > \mu(v_n)$
then compute $\boldsymbol{u}_n^A$ (2.2);
update $\boldsymbol{u}_n := \boldsymbol{u}_n^A$;
compute the coordinates for moving to the next step $P\colon x_n$ (1.1);
compute the distance between players $|z_n|$;
update the step counter $n := n + 1$;
return to Step 4;
else compute $\boldsymbol{u}_n^F$ (2.3);
update $\boldsymbol{u}_n := \boldsymbol{u}_n^F$;
compute capture point coordinates $P\colon x_n$ (1.1);
output the capture step n, the GPS N, the capture point $P\colon x_n$.

Step 8. End. (Block diagram for DPP shown in Fig. 1)

Example 4.1. Based on the obtained theoretical results and the constructed algorithm, an animation model for the discrete pursuit problem is implemented. The animation model demonstrates that, when applying the Π_i-strategy, the players' traces converge step by step and in parallel, which confirms the validity of the theoretical results. The model provides not only a graphical illustration but also the possibility of presenting a numerical data table for each step, which makes it possible to clearly analyze the results when using the Π-strategy (see Fig. 2). The example is given for the initial parameters specified in Table 1.

Table 1. Initial parameters for the pursuit–evasion simulation

Parameters	Descriptions	Values
x_0	Initial position of the pursuer	$(-16, -14)$
y_0	Initial position of the evader	$(1, 1)$
α	Initial resource of the pursuer	2
β	Initial resource of the evader	1,1
$v_1 = (v_{1_1}, v_{2_1})$	Initial control vector of the evader	$\left(-\sin i, -\frac{1}{1+i}\right)$
Strategy	Pursuer's control strategy	Π-strategy
Simulation step	Theoretically guaranteed upper bound on capture step	26

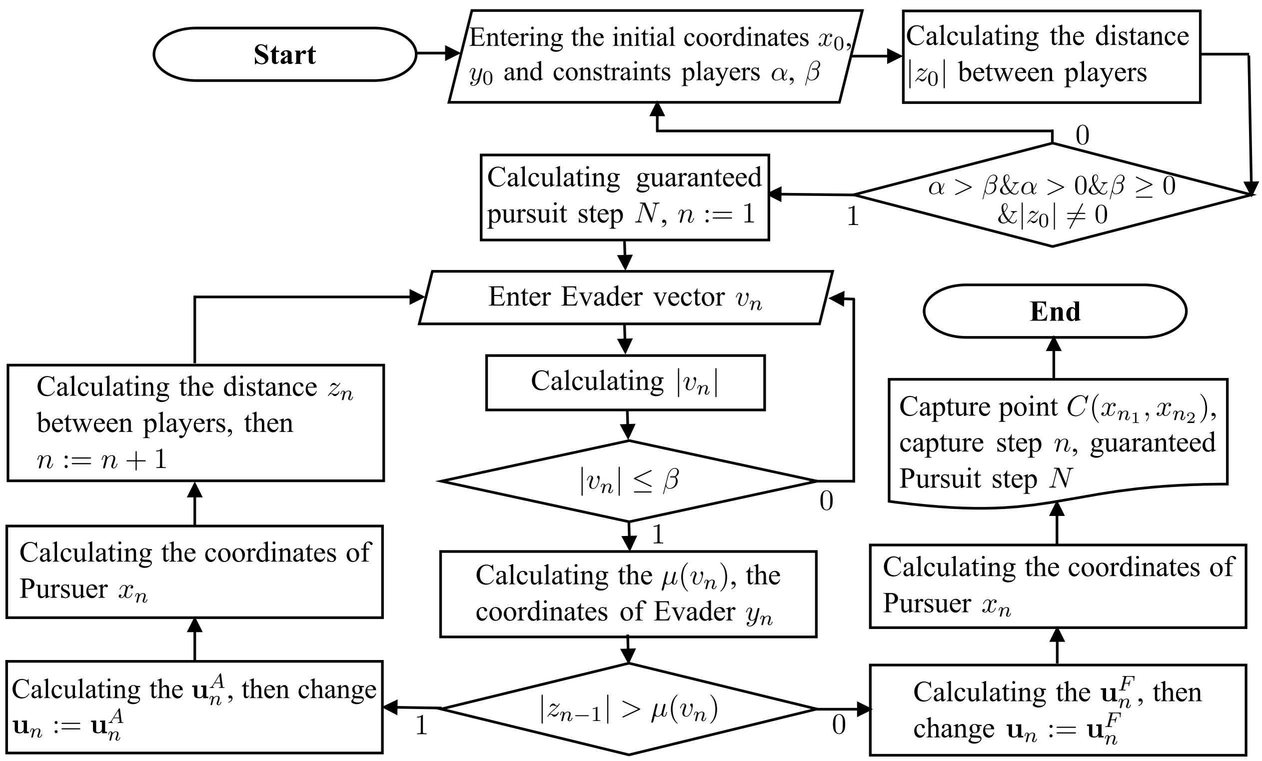


Fig. 1. Block diagram for DPP

4.2. Group Discrete Pursuit Problem: algorithm and implementation

This subsection proposes a software algorithm for solving the pursuit problem involving three pursuers P^1, P^2, P^3 and one evader E. Note that the structure of the GDPP algorithm (3.1)–(3.2) coincides with that of the DPP algorithm (1.1)–(1.2). In equations (3.1)–(3.2), it is assumed that the constraints for all players are equal and normalized to one. In the developed GDPP algorithm, the constraint for the evader is defined by the parameter β, which is set equal to α_i for each of the three pursuers, where $i = 1, 2, 3$.

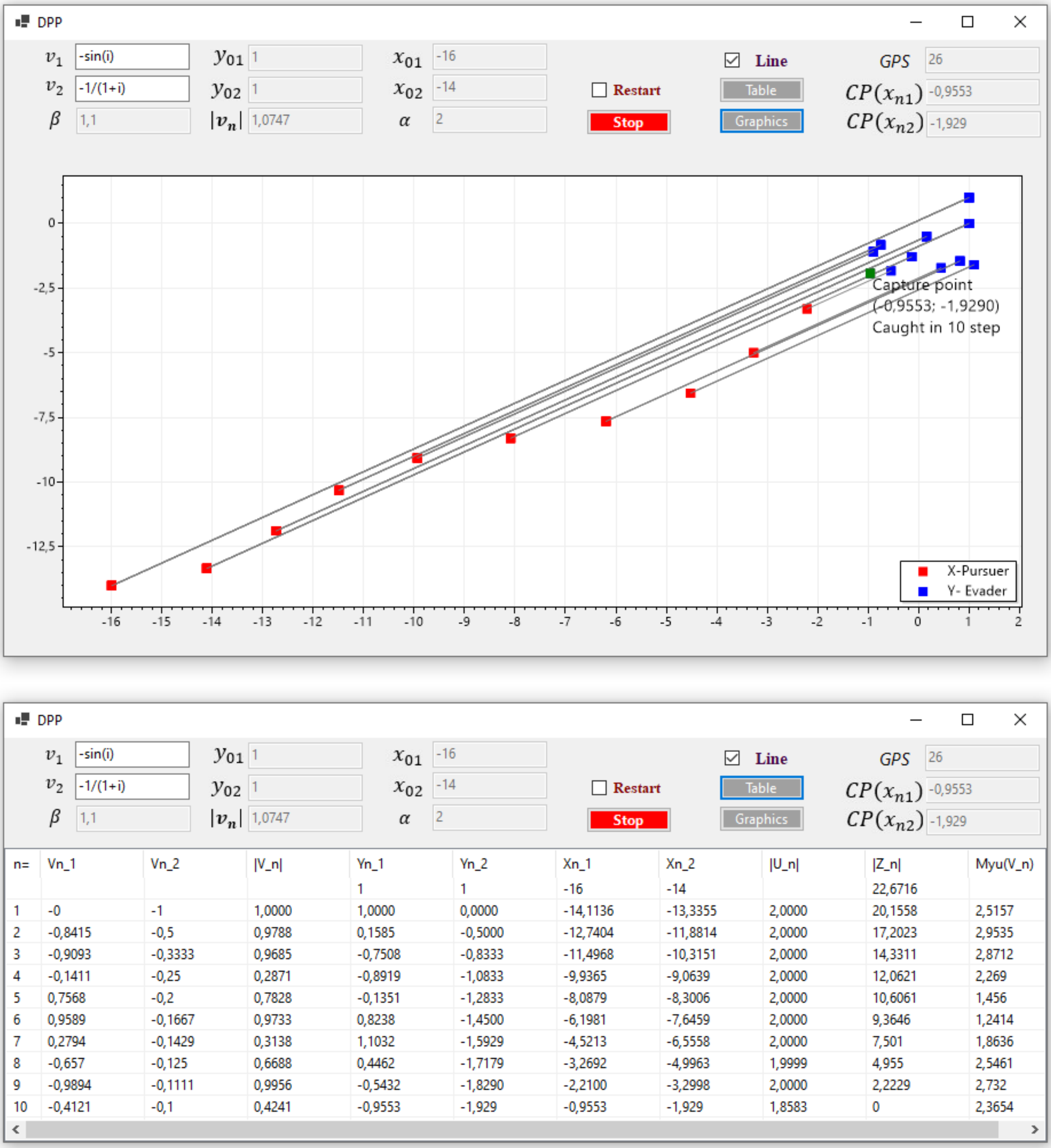
Step 1. Enter the data for the pursuers P^i, $i = 1, 2, 3$, and the evader E, including the coordinates of their initial states $x_0^i := (x_{01}^i, x_{02}^i)$, $y_0 := (y_{01}, y_{02})$, along with the control constraint parameter β.

Step 2. **If** $(\beta \geqslant 0)$
then set $\alpha_1 := \alpha_2 := \alpha_3 := \beta$;
else return to Step 1.

Step 3. **for** $i := 1$ **to** 3 **do** perform the following calculations:
calculate the distance between players:
$z_{01}^i := x_{01}^i - y_{01}$, $z_{02}^i := x_{02}^i - y_{02}$, $|z_0^i| := \sqrt{(z_{01}^i)^2 + (z_{02}^i)^2}$;
check condition: **If** $(|z_0^i| \neq 0)$
then compute: $\xi_{01}^i := \frac{z_{01}^i}{|z_0^i|}$, $\xi_{02}^i := \frac{z_{02}^i}{|z_0^i|}$
else return to Step 1.

Step 4. The program performs the following calculations:
computes the determinants:

$$d_1 := \begin{vmatrix} \xi_{01}^1 & \xi_{01}^2 \\ \xi_{02}^1 & \xi_{02}^2 \end{vmatrix}, \quad d_2 := \begin{vmatrix} \xi_{01}^2 & \xi_{01}^3 \\ \xi_{02}^2 & \xi_{02}^3 \end{vmatrix}, \quad d_3 := \begin{vmatrix} \xi_{01}^3 & \xi_{01}^1 \\ \xi_{02}^3 & \xi_{02}^1 \end{vmatrix}. \tag{4.1}$$



n=	Vn_1	Vn_2	|V_n|	Yn_1	Yn_2	Xn_1	Xn_2	|U_n|	|Z_n|	Myu(V_n)
				1	1	-16	-14		22,6716	
1	-0	-1	1,0000	1,0000	0,0000	-14,1136	-13,3355	2,0000	20,1558	2,5157
2	-0,8415	-0,5	0,9788	0,1585	-0,5000	-12,7404	-11,8814	2,0000	17,2023	2,9535
3	-0,9093	-0,3333	0,9685	-0,7508	-0,8333	-11,4968	-10,3151	2,0000	14,3311	2,8712
4	-0,1411	-0,25	0,2871	-0,8919	-1,0833	-9,9365	-9,0639	2,0000	12,0621	2,269
5	0,7568	-0,2	0,7828	-0,1351	-1,2833	-8,0879	-8,3006	2,0000	10,6061	1,456
6	0,9589	-0,1667	0,9733	0,8238	-1,4500	-6,1981	-7,6459	2,0000	9,3646	1,2414
7	0,2794	-0,1429	0,3138	1,1032	-1,5929	-4,5213	-6,5558	2,0000	7,501	1,8636
8	-0,657	-0,125	0,6688	0,4462	-1,7179	-3,2692	-4,9963	1,9999	4,955	2,5461
9	-0,9894	-0,1111	0,9956	-0,5432	-1,8290	-2,2100	-3,2998	2,0000	2,2229	2,732
10	-0,4121	-0,1	0,4241	-0,9553	-1,929	-0,9553	-1,929	1,8583	0	2,3654

Fig. 2. The DPP animation model and its data table

Step 5. **If**

$$(d_1 > 0 \wedge d_2 > 0 \wedge d_3 > 0) \vee (d_1 < 0 \wedge d_2 < 0 \wedge d_3 < 0) \tag{4.2}$$

then execute the Procedure: *Calculation of the Guaranteed Pursuit Step (GPS)*;
else *the evader has a chance to escape the pursuit*
If “Press Stop”
then go to Step 10;
else set $n := 1$.

Note that for the case under consideration, conditions (3.11) and (4.2) are equivalent, which easily follows from the properties of the oriented vectors $\xi_0^1 = (\xi_{01}^1, \xi_{02}^1)$, $\xi_0^2 = (\xi_{01}^2, \xi_{02}^2)$ and $\xi_0^3 = (\xi_{01}^3, \xi_{02}^3)$, that is, if the signs of all determinants (4.1) are simultaneously positive or negative, then the initial state of the evader is inside the convex hull of the initial states of the pursuers (Fig. 5).

Step 6. Enter the evader vector $v_n := (v_{n1}, v_{n2})$.

Step 7. Compute $|v_n| := \sqrt{v_{n1}^2 + v_{n2}^2}$.

Step 8. **If** $|v_n| \leqslant \beta$
then compute the evader’s position $E\colon y_n$ (3.2);
else return to Step 6.

Step 9. **for** $i := 1$ **to** 3 **do** perform the following calculations:
compute $\mu_i(v_n)$ (3.10)
check condition:
If $|z_{n-1}^i| > \mu_i(v_n)$
then compute $\boldsymbol{u}_n^{A_i}$ (3.8);
update $\boldsymbol{u}_n^i := \boldsymbol{u}_n^{A_i}$;
compute the coordinates for moving to the next step $P^i\colon x_n^i$ (3.1);
compute the distance between players $|z_n^i|$;
update the step counter $n := n + 1$; return to Step 6;
else compute $\boldsymbol{u}_n^{F_i}$ (3.9);
set $\boldsymbol{u}_n^i := \boldsymbol{u}_n^{F_i}$;
compute $P^i\colon x_n^i$ (3.1);
output the capture step n, the GPS N, the capture points $P^i\colon x_n^i$.

Step 10. End. (Block diagram for GDPP shown in Fig. 3).

Procedure: Calculation of the Guaranteed Pursuit Step (GPS).

Step 1. Calculation of the value δ (Minimax Calculation).
The segment $[0, 2\pi]$ is uniformly divided into 1000 equal parts, where θ is defined as:

$$\theta_k := \frac{2\pi(k-1)}{1000}, \quad k = 1, 2, \ldots, 1000.$$

For each direction θ_k, the unit vector components are calculated as:

$$p_{k1} := \cos(\theta_k), \quad p_{k2} := \sin(\theta_k).$$

Then, for each k, the following values are computed:

$$q_k := \max_{i=1,2,3} \left(p_{k1}\xi_{01}^i + p_{k2}\xi_{02}^i\right),$$

and the value δ is determined as $\delta := \min\limits_{k=1,2,\ldots,1000} q_k$.

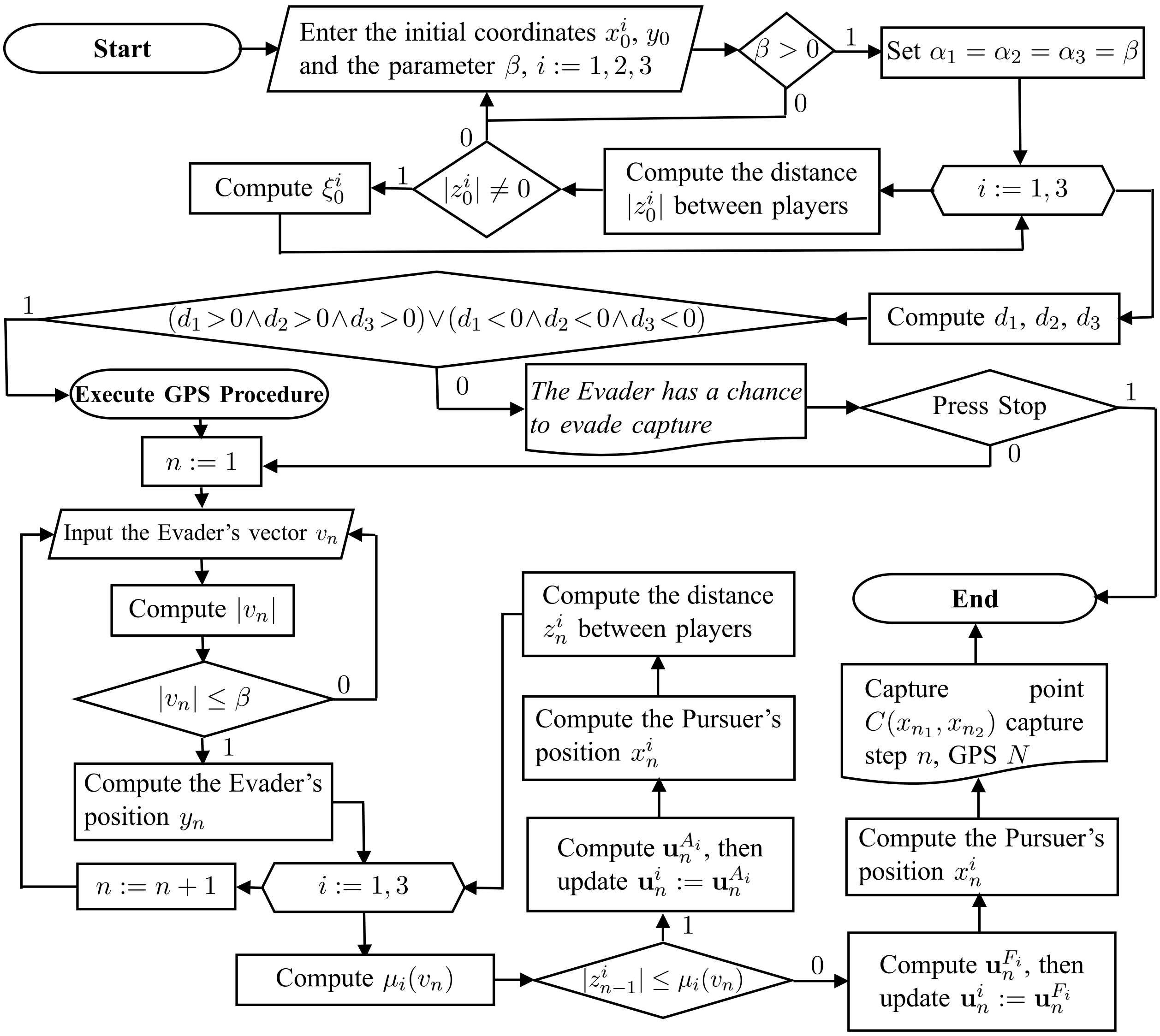


Fig. 3. Block diagram for GDPP

Step 2. Calculation of the Guaranteed Pursuit Step N. The GPS value N is computed using the formula:

$$N := \left[\sum_{i=1}^{3} |z_0^i|/\gamma \right] + 1, \quad \gamma = \frac{2\delta}{1 + 2\delta}.$$

(Block diagram for GPS shown in Fig. 4).

Example 4.2. Based on the obtained theoretical results and the constructed algorithm, an animation model is implemented for the case of three pursuers and one evader. The model performs a two-stage procedure: first, it checks whether the evader lies within the convex hull of the pursuers' direction vectors; then it computes the minimax estimate δ, after which the resulting GPS value is displayed in the interface. When applying the Π_i-strategy, the animation model demonstrates the step-by-step parallel convergence of the players' traces, which confirms the validity of the theoretical results. The model provides not only a graphical illustration but also the possibility of presenting a numerical data table for each step, which makes it possible to clearly analyze the results when using the Π_i-strategy (see Fig. 5). The example is presented for the initial parameters specified in Table 2.

Table 2. Initial parameters for the pursuit–evasion simulation

Parameters	Descriptions	Values
x_0^1	Initial position of the pursuer P^1	$(2, 12)$
x_0^2	Initial position of the pursuer P^2	$(3, -7)$
x_0^3	Initial position of the pursuer P^3	$(-3, 11)$
y_0	Initial position of the evader	$(1, 2)$
β	Initial resource of the evader	1
$\alpha_1, \alpha_2, \alpha_3$	Initial resource of the pursuer	1
$v_1 = (v_{1_1}, v_{2_1})$	Initial control vector of the evader	$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$
Strategy	Pursuer's control strategy	Π_i-strategy
Simulation step	Theoretically guaranteed upper bound on capture step	174

Conclusion

For both DPP and GDPP, animation models were developed using Visual $C\#$ and the ScottPlot.WinForms and Zirpl.CalcEngine technologies. The animation models demonstrate the step-by-step application of the pursuers' Π-strategy, allowing clear visualization of the players' traces converging. In addition to the graphical visualization, the models provide numerical data tables for verifying the correctness of the obtained results. The programs are hosted on Google Drive, which allows testing their functionality and using them for further analysis:

a) the Animation Model Program for the Discrete Pursuit Problem (DPP)
https://drive.google.com/file/d/1ZbZ03t8UGf1_rw8iYwFVMJR83B7S8bSY/view?usp=drive_link;

b) the Animation Model for the Group Discrete Pursuit Problem (GDPP)
https://drive.google.com/file/d/1ZxOJ7AybnVu_AvDBQ9EH0B_WFJmMEht1/view?usp=drive_link, which facilitates testing the model's functionality.

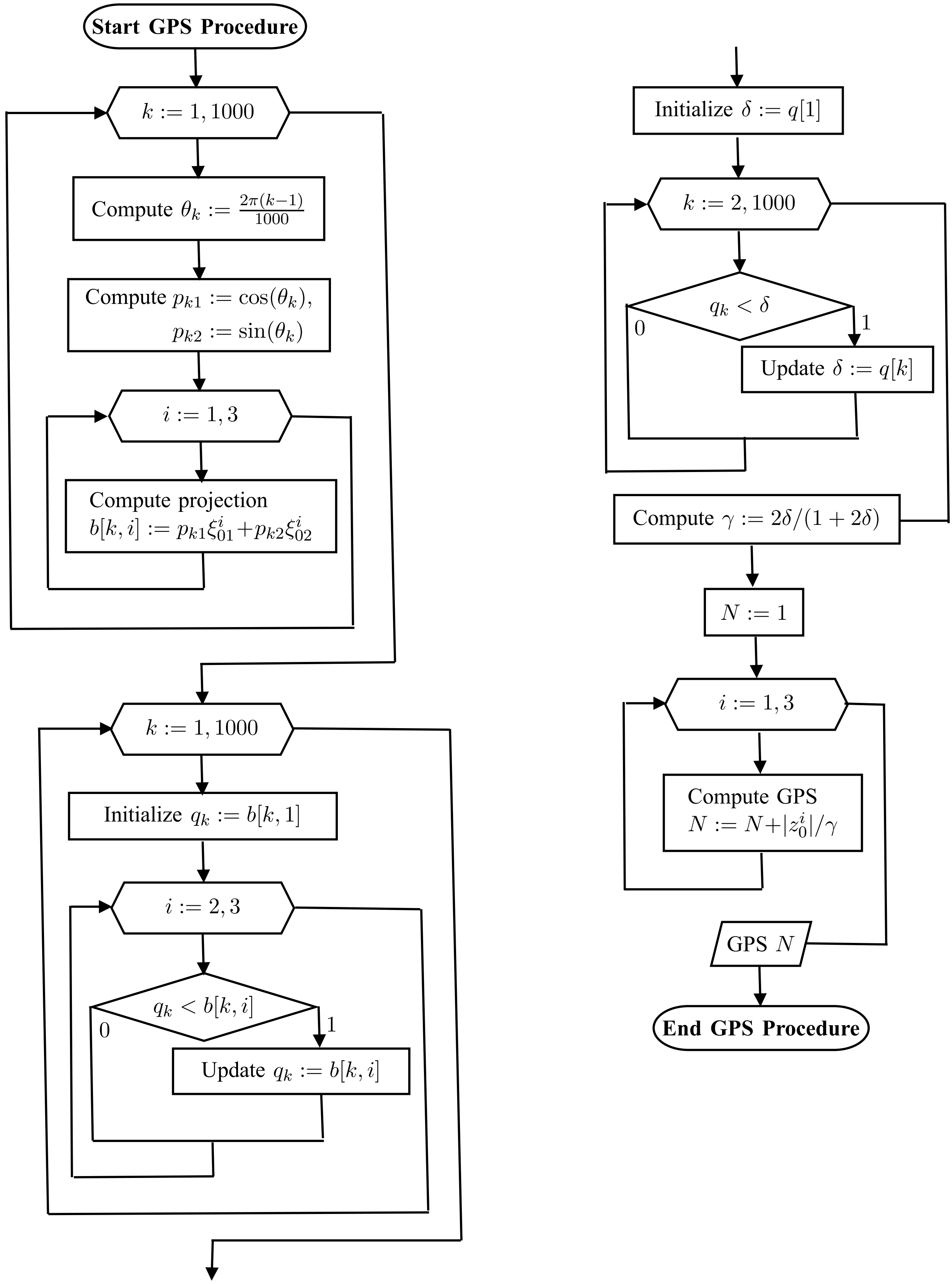
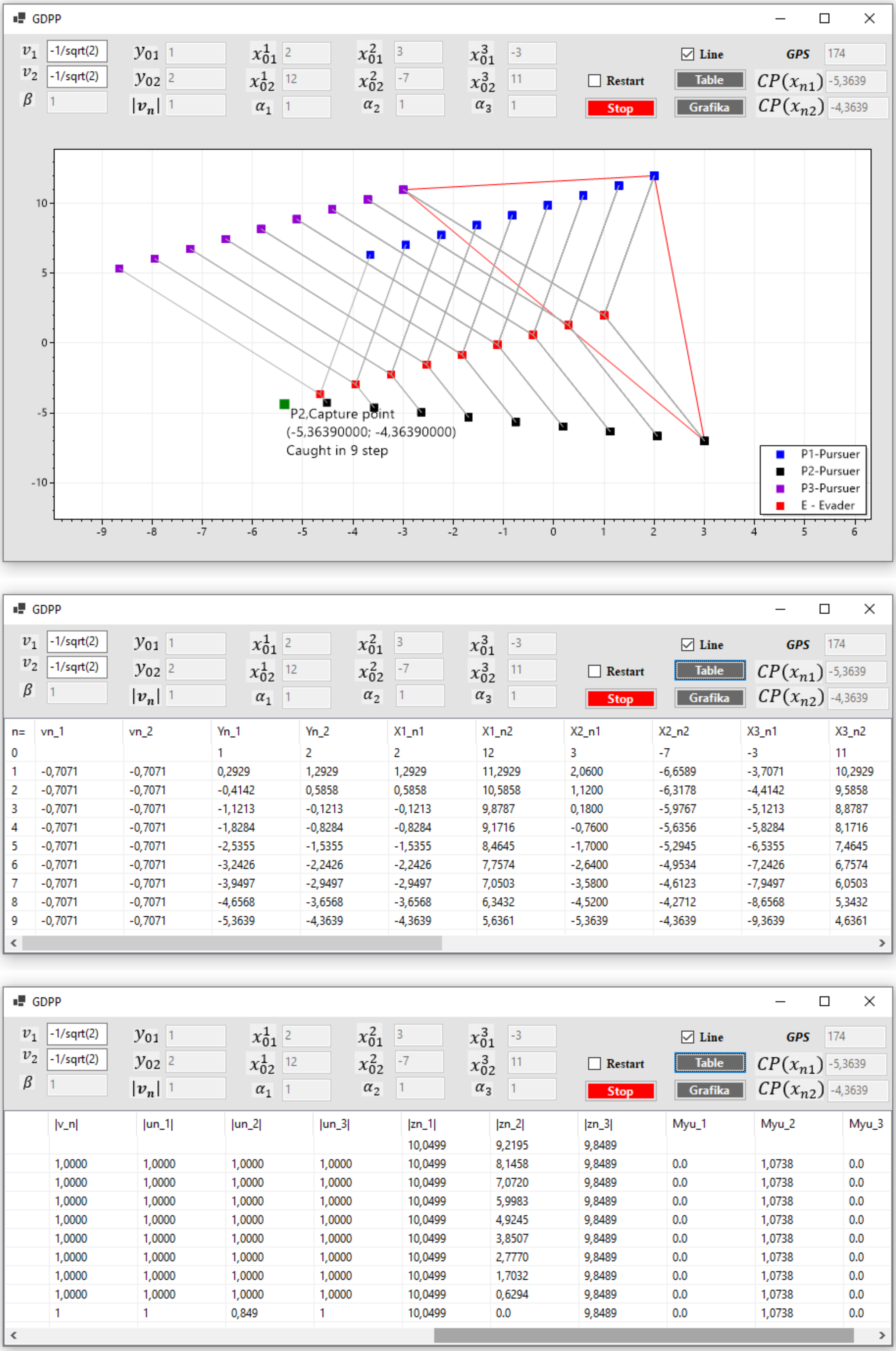


Fig. 4. Block diagram for GPS Procedure



n=	vn_1	vn_2	Yn_1	Yn_2	X1_n1	X1_n2	X2_n1	X2_n2	X3_n1	X3_n2
0			1	2	2	12	3	-7	-3	11
1	-0,7071	-0,7071	0,2929	1,2929	1,2929	11,2929	2,0600	-6,6589	-3,7071	10,2929
2	-0,7071	-0,7071	-0,4142	0,5858	0,5858	10,5858	1,1200	-6,3178	-4,4142	9,5858
3	-0,7071	-0,7071	-1,1213	-0,1213	-0,1213	9,8787	0,1800	-5,9767	-5,1213	8,8787
4	-0,7071	-0,7071	-1,8284	-0,8284	-0,8284	9,1716	-0,7600	-5,6356	-5,8284	8,1716
5	-0,7071	-0,7071	-2,5355	-1,5355	-1,5355	8,4645	-1,7000	-5,2945	-6,5355	7,4645
6	-0,7071	-0,7071	-3,2426	-2,2426	-2,2426	7,7574	-2,6400	-4,9534	-7,2426	6,7574
7	-0,7071	-0,7071	-3,9497	-2,9497	-2,9497	7,0503	-3,5800	-4,6123	-7,9497	6,0503
8	-0,7071	-0,7071	-4,6568	-3,6568	-3,6568	6,3432	-4,5200	-4,2712	-8,6568	5,3432
9	-0,7071	-0,7071	-5,3639	-4,3639	-4,3639	5,6361	-5,3639	-4,3639	-9,3639	4,6361

|v_n|	|un_1|	|un_2|	|un_3|	|zn_1|	|zn_2|	|zn_3|	Myu_1	Myu_2	Myu_3
				10,0499	9,2195	9,8489			
1,0000	1,0000	1,0000	1,0000	10,0499	8,1458	9,8489	0.0	1,0738	0.0
1,0000	1,0000	1,0000	1,0000	10,0499	7,0720	9,8489	0.0	1,0738	0.0
1,0000	1,0000	1,0000	1,0000	10,0499	5,9983	9,8489	0.0	1,0738	0.0
1,0000	1,0000	1,0000	1,0000	10,0499	4,9245	9,8489	0.0	1,0738	0.0
1,0000	1,0000	1,0000	1,0000	10,0499	3,8507	9,8489	0.0	1,0738	0.0
1,0000	1,0000	1,0000	1,0000	10,0499	2,7770	9,8489	0.0	1,0738	0.0
1,0000	1,0000	1,0000	1,0000	10,0499	1,7032	9,8489	0.0	1,0738	0.0
1,0000	1,0000	1,0000	1,0000	10,0499	0,6294	9,8489	0.0	1,0738	0.0
1	1	0,849	1	10,0499	0.0	9,8489	0.0	1,0738	0.0

Fig. 5. The GDPP animation model and its data table

REFERENCES

1. Isaacs R. *Differential games*, New York: John Wiley and Sons, 1965. https://zbmath.org/0125.38001
2. Berkovitz L. D. A variational approach to differential games, *Advances in game theory*, Princeton: Princeton University Press, 1964, pp. 127–174. https://doi.org/10.1515/9781400882014-011
3. Fleming W. H. The convergence problem for differential games, II, *Advances in game theory*, Princeton: Princeton University Press, 1964, pp. 195–210. https://doi.org/10.1515/9781400882014-013
4. Friedman A. *Differential games*, Courier Corporation, 2013.
5. Pontryagin L. S. *Izbrannye nauchnye trudy. Tom II: Differentsial'nye uravneniya. Teoriya operatorov. Optimal'noe upravlenie. Differentsial'nye igry* (Selected scientific works. Vol. II: Differential equations. Operator theory. Optimal control. Differential games), Moscow: Nauka, 1988. https://zbmath.org/0656.01015
6. Krasovskii N. N., Subbotin A. I. *Game-theoretical control problems*, New York: Springer, 1988. https://www.springer.com/gp/book/9781461283188
7. Azamov A. A. *Osnovaniya teorii diskretnykh igr* (Fundamentals of theory of discrete games), Tashkent: Niso Poligraf, 2011.
8. Azamov A. A., Kuchkarov A. Sh. On controllability and pursuit problems in linear discrete systems, *Journal of Computer and Systems Sciences International*, 2010, vol. 49, issue 3, pp. 360–365. https://doi.org/10.1134/S1064230710030044
9. Ibragimov G. I. Problems of linear discrete games of pursuit, *Mathematical Notes*, 2005, vol. 77, no. 5, pp. 653–662. https://doi.org/10.1007/s11006-005-0065-4
10. Krivonos Yu. G., Matichin I. I., Chikrii A. A. *Dinamicheskie igry s razryvnymi traektoriyami* (Dynamic games with discontinuous paths), Kiev: Naukova dumka, 2005.
11. Kuchkarov A., Ibragimov G., Sotvoldiev A. Linear discrete pursuit game problem with total constraints, *Abstract and Applied Analysis*, 2013, vol. 2013. https://doi.org/10.1155/2013/840925
12. Aulbach B., Hilger S. Linear dynamic processes with inhomogeneous time scale, *Nonlinear dynamics and quantum dynamical systems*, De Gruyter, 1990, pp. 9–20. https://doi.org/10.1515/9783112581445-002
13. Hilger S. Analysis on measure chains — a unified approach to continuous and discrete calculus, *Results in Mathematics*, 1990, vol. 18, nos. 1–2, pp. 18–56. https://doi.org/10.1007/BF03323153
14. Bohner M., Peterson A. *Advances in dynamic equations on time scales*, Boston: Birkhäuser, 2003. https://doi.org/10.1007/978-0-8176-8230-9
15. Martins N., Torres D. F. M. Necessary conditions for linear noncooperative N-player delta differential games on time scales, *Discussiones Mathematicae. Differential Inclusions, Control and Optimization*, 2011, vol. 31, no. 1, pp. 23–37. https://doi.org/10.7151/dmdico.1126
16. Petrov N. N. The problem of simple group pursuit with phase constraints in time scales, *Vestnik Udmurtskogo Universiteta. Matematika. Mekhanika. Komp'yuternye Nauki*, 2020, vol. 30, issue 2, pp. 249–258 (in Russian). https://doi.org/10.35634/vm200208
17. Petrov N. N. Multiple capture of a given number of evaders in the problem of simple pursuit with phase restrictions on timescales, *Dynamic Games and Applications*, 2022, vol. 12, no. 2, pp. 632–642. https://doi.org/10.1007/s13235-021-00387-y
18. Petrov N. N., Mozhegova E. S. Simple pursuit problem with phase constraints of two coordinated evaders on time scales, *Doklady Mathematics*, 2023, vol. 108, no. S1, pp. S86–S91. https://doi.org/10.1134/S1064562423600720
19. Pshenichnyi B. N. Simple pursuit by several objects, *Cybernetics*, 1976, vol. 12, no. 3, pp. 484–485. https://doi.org/10.1007/bf01070036
20. Petrosjan L. A. *Differential games of pursuit*, Singapore: World Scientific, 1993. https://doi.org/10.1142/1670
21. Blagodatskikh A. I., Petrov N. N. *Konfliktnoe vzaimodeistvie grupp upravlyaemykh ob"ektov* (Conflict interaction of groups of controlled objects), Izhevsk: Udmurt State University, 2009.
22. Satimov N. Yu. *Metody resheniya zadachi presledovaniya v teorii differentsial'nykh igr* (Methods of solving of pursuit problem in differential games), Tashkent: National University of Uzbekistan, 2003.

23. Chikrii A. *Conflict-controlled processes*, Dordrecht: Springer, 1997. https://doi.org/10.1007/978-94-017-1135-7
24. Grigorenko N. L. *Matematicheskie metody upravleniya neskol'kimi dinamicheskimi protsessami* (Mathematical methods for control of several dynamic processes), Moscow: Moscow State University, 1990.
25. Azamov A. On the quality problem for simple pursuit games with constraint, *Serdica. Bulgaricae matematicae publicationes*, 1986, vol. 12, no. 1, pp. 38–43 (in Russian). https://zbmath.org/0629.90107
26. Azamov A. A., Samatov B. T. The Π-strategy: analogies and applications, *Contributions to Game Theory and Management*, 2011, vol. 4, pp. 33–46. https://www.mathnet.ru/eng/cgtm177
27. Samatov B. T. On a pursuit–evasion problem under a linear change of the pursuer resource, *Siberian Advances in Mathematics*, 2013, vol. 23, no. 10, pp. 294–302. https://doi.org/10.3103/S1055134413040056
28. Samatov B. T. The pursuit–evasion problem under integral–geometric constraints on pursuer controls, *Automation and Remote Control*, 2013, vol. 74, no. 7, pp. 1072–1081. https://doi.org/10.1134/S0005117913070023
29. Samatov B. T. The Π-strategy in a differential game with linear control constraints, *Journal of Applied Mathematics and Mechanics*, 2014, vol. 78, issue 3, pp. 258–263. https://doi.org/10.1016/j.jappmathmech.2014.09.008
30. Samatov B., Umaraliyeva N. The Π-strategy and its animation model in plane, *AIP Conference Proceedings*, 2024, vol. 3244, issue 1, 020026. https://doi.org/10.1063/5.0241699
31. Samatov B. T., Umaraliyeva N. T., Xolmirzayev X., Abduraxmanov A. *Animatsionnaya model' zadachi presledovaniya–ubeganiya pri geometricheskikh ogranicheniyakh* (Animation model of the pursuit–evasion problem with geometric constraints), Uzbekistan Patent, 2023, no. DGU 27303. https://im.adliya.uz/document/check/9f7633ba-2746-4724-ad64-fdbfd9644984
32. Samatov B. T., Umaraliyeva N. T., Xolmirzayev X., Soyibboyev O. *Animatsionnaya model' zadachi gruppovogo presledovaniya s geometricheskimi ogranicheniyami na elementy upravleniya igrokami* (Animation model of the group pursuit problem with geometric constraints on the players' control), Uzbekistan Patent, 2023, no. DGU 27302. https://im.adliya.uz/document/check/87ac0834-ec72-410e-bd28-8b795eeebbbd

Received 03.06.2025
Accepted 20.01.2026

Abdulla Azamovich Azamov, Doctor of Physics and Mathematics, Professor, Laboratory of Dynamical Systems and Their Applications, V. I. Romanovsky Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan, University str., 9, Tashkent, 100174, Uzbekistan.
ORCID: https://orcid.org/0000-0003-3516-0904
E-mail: abdulla.azamov@gmail.com

Bahrom Tadjiahmatovich Samatov, Doctor of Physics and Mathematics, Professor, Laboratory of Dynamical Systems and Their Applications, V. I. Romanovsky Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan, University str., 9, Tashkent, 100174, Uzbekistan.
ORCID: https://orcid.org/0000-0002-0734-8507
E-mail: samatov57@gmail.com

Nargiza Tashkinbayevna Umaraliyeva, PhD student, Department of Applied Mathematics, Namangan State University, Uychi str., 316, Namangan, 116019, Uzbekistan.
ORCID: https://orcid.org/0009-0003-1855-364X
E-mail: umaraliyeva80@gmail.com

Citation: A. A. Azamov, B. T. Samatov, N. T. Umaraliyeva. Π-strategy algorithm in discrete pursuit games, *Vestnik Udmurtskogo Universiteta. Matematika. Mekhanika. Komp'yuternye Nauki*, 2026, vol. 36, issue 1, pp. 3–22.

МАТЕМАТИКА 2026. Т. 36. Вып. 1. С. 3–22.

А. А. Азамов, Б. Т. Саматов, Н. Т. Умаралиева

Алгоритм Π-стратегии в дискретных играх преследования

Ключевые слова: дискретная игра, преследователь, убегающий, стратегия, преследования, гарантированный шаг поимки.

УДК 517.977

DOI: 10.35634/vm260101

Данная работа посвящена исследованию одной известной задачи Б. Н. Пшеничного, а именно задаче простого группового преследования, когда игроки совершают пошаговые перемещения. В работе рассматриваются два отдельных случая. В первом случае решается дискретная игра преследования, когда в игре участвуют только один преследователь и один убегающий. Для решения этой задачи приводится алгоритм применения Π-стратегии. Согласно предлагаемому методу, игроки сначала сближаются, и в итоге точно совпадают. Во втором случае предлагаемый метод решения распространяется на игру группового преследования. Полученные результаты проверяются с помощью анимационных моделей, созданных на языке программирования Visual $C\#$ с использованием технологии ScottPlot.WinForms.

СПИСОК ЛИТЕРАТУРЫ

1. Айзекс Р. Дифференциальные игры. М.: Мир, 1967.
2. Berkovitz L. D. A variational approach to differential games // Advances in game theory. Princeton: Princeton University Press, 1964. P. 127–174. https://doi.org/10.1515/9781400882014-011
3. Fleming W. H. The convergence problem for differential games, II // Advances in game theory. Princeton: Princeton University Press, 1964. P. 195–210. https://doi.org/10.1515/9781400882014-013
4. Friedman A. Differential games. Courier Corporation, 2013.
5. Понтрягин Л. С. Избранные научные труды. Т. 2. Дифференциальные уравнения. Теория операторов. Оптимальное управление. Дифференциальные игры. М.: Наука, 1988. https://zbmath.org/0656.01015
6. Красовский Н. Н., Субботин А. И. Позиционные дифференциальные игры. М.: Наука, 1974.
7. Азамов А. А. Основания теории дискретных игр. Ташкент: Niso Poligraf, 2011.
8. Азамов А. А., Кучкаров А. Ш. О задачах управляемости и преследования в линейных дискретных системах // Известия Российской академии наук. Теория и системы управления. 2010. № 3. С. 21–26. https://elibrary.ru/item.asp?id=16455062
9. Ибрагимов Г. И. О задачах линейной дискретной игры преследования // Математические заметки. 2005. Т. 77. Вып. 5. С. 707–718. https://doi.org/10.4213/mzm2526
10. Кривонос Ю. Г., Матичин И. И., Чикрий А. А. Динамические игры с разрывными траекториями. Киев: Наукова думка, 2005.
11. Kuchkarov A., Ibragimov G., Sotvoldiev A. Linear discrete pursuit game problem with total constraints // Abstract and Applied Analysis. 2013. Vol. 2013. https://doi.org/10.1155/2013/840925
12. Aulbach B., Hilger S. Linear dynamic processes with inhomogeneous time scale // Nonlinear dynamics and quantum dynamical systems. De Gruyter, 1990. P. 9–20. https://doi.org/10.1515/9783112581445-002
13. Hilger S. Analysis on measure chains — a unified approach to continuous and discrete calculus // Results in Mathematics. 1990. Vol. 18. Nos. 1–2. P. 18–56. https://doi.org/10.1007/BF03323153
14. Bohner M., Peterson A. Advances in dynamic equations on time scales. Boston: Birkhäuser, 2003. https://doi.org/10.1007/978-0-8176-8230-9
15. Martins N., Torres D. F. M. Necessary conditions for linear noncooperative N-player delta differential games on time scales // Discussiones Mathematicae. Differential Inclusions, Control and Optimization. 2011. Vol. 31. No. 1. P. 23–37. https://doi.org/10.7151/dmdico.1126

16. Петров Н. Н. Задача простого группового преследования с фазовыми ограничениями во временных шкалах // Вестник Удмуртского университета. Математика. Механика. Компьютерные науки. 2020. Т. 30. Вып. 2. С. 249–258. https://doi.org/10.35634/vm200208
17. Petrov N. N. Multiple capture of a given number of evaders in the problem of simple pursuit with phase restrictions on timescales // Dynamic Games and Applications. 2022. Vol. 12. No. 2. P. 632–642. https://doi.org/10.1007/s13235-021-00387-y
18. Petrov N. N., Mozhegova E. S. Simple pursuit problem with phase constraints of two coordinated evaders on time scales // Doklady Mathematics. 2023. Vol. 108. No. S1. P. S86–S91. https://doi.org/10.1134/S1064562423600720
19. Пшеничный Б. Н. Простое преследование несколькими объектами // Кибернетика. 1976. Вып. 3. С. 145–146.
20. Petrosjan L. A. Differential games of pursuit. Singapore: World Scientific, 1993. https://doi.org/10.1142/1670
21. Благодатских А. И., Петров Н. Н. Конфликтное взаимодействие групп управляемых объектов. Ижевск: УдГУ, 2009.
22. Сатимов Н. Ю. Методы решения задачи преследования в теории дифференциальных игр. Ташкент: Изд-во НУУз, 2003.
23. Чикрий А. А. Конфликтно управляемые процессы. Киев: Наукова думка, 1992.
24. Григоренко Н. Л. Математические методы управления несколькими динамическими процессами. М.: Изд-во Моск. ун-та, 1990.
25. Азамов А. А. О задаче качества для игр простого преследования с ограничением // Сердика. Българско математическо списание. 1986. Т. 12. Вып. 1. С. 38–43. https://zbmath.org/0629.90107
26. Azamov A. A., Samatov B. T. The П-strategy: analogies and applications // Contributions to Game Theory and Management. 2011. Vol. 4. P. 33–46. https://www.mathnet.ru/rus/cgtm177
27. Саматов Б. Т. О задаче преследования–убегания при линейном изменении ресурса преследователя // Математические труды. 2012. Т. 15. Вып. 2. С. 159–171.
28. Саматов Б. Т. Задача преследования–убегания при интегрально-геометрических ограничениях на управления преследователя // Автоматика и телемеханика. 2013. Вып. 7. С. 17–28. https://www.mathnet.ru/rus/at5465
29. Саматов Б. Т. П-стратегия в дифференциальной игре с линейными ограничениями по управлению // Прикладная математика и механика. 2014. Т. 78. Вып. 3. С. 369–377. https://pmm.ipmnet.ru/ru/Issues/2014/78-3/369
30. Samatov B., Umaraliyeva N. The П-strategy and its animation model in plane // AIP Conference Proceedings. 2024. Vol. 3244. Issue 1. 020026. https://doi.org/10.1063/5.0241699
31. Саматов Б. Т., Умаралиева Н. Т., Холмирзаев Х., Абдурахманов А. Анимационная модель задачи преследования–убегания при геометрических ограничениях // Патент Узбекистана. 2023. № DGU 27303. https://im.adliya.uz/document/check/9f7633ba-2746-4724-ad64-fdbfd9644984
32. Саматов Б. Т., Умаралиева Н. Т., Холмирзаев Х., Сойиббоев У. Анимационная модель задачи группового преследования с геометрическими ограничениями на элементы управления игроками // Патент Узбекистана. 2023. № DGU 27302. https://im.adliya.uz/document/check/87ac0834-ec72-410e-bd28-8b795eeebbbd

Поступила в редакцию 03.06.2025
Принята к публикации 20.01.2026

Азамов Абдулла Азамович, д. ф.-м. н., профессор, лаборатория динамических систем и их применений, Институт математики им. В. И. Романовского Академии наук Республики Узбекистан, 100174, Узбекистан, г. Ташкент, ул. Университетская, 9.
ORCID: https://orcid.org/0000-0003-3516-0904
E-mail: abdulla.azamov@gmail.com

Саматов Бахром Таджиахматович, д. ф.-м. н., профессор, лаборатория динамических систем и их применений, Институт математики им. В. И. Романовского Академии наук Республики Узбекистан, 100174, Узбекистан, г. Ташкент, ул. Университетская, 9.
ORCID: https://orcid.org/0000-0002-0734-8507
E-mail: samatov57@gmail.com

Умаралиева Наргиза Ташкинбаевна, аспирант, кафедра прикладной математики, Наманганский государственный университет, 116019, Узбекистан, г. Наманган, ул. Уйчи, 316.
ORCID: https://orcid.org/0009-0003-1855-364X
E-mail: umaraliyeva80@gmail.com

Цитирование: А. А. Азамов, Б. Т. Саматов, Н. Т. Умаралиева. Алгоритм П-стратегии в дискретных играх преследования // Вестник Удмуртского университета. Математика. Механика. Компьютерные науки. 2026. Т. 36. Вып. 1. С. 3–22.