VESTNIK UDMURTSKOGO UNIVERSITETA. MATEMATIKA. MEKHANIKA. KOMP'YUTERNYE NAUKI
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SIMPLE GROUP PURSUIT WITH FRACTIONAL DERIVATIVES AND INTEGRAL CONSTRAINTS

This paper addresses the problem of simple pursuit of one evader by a group of pursuers in a differential game described by equations with Caputo fractional derivatives from the interval $(0, 1)$. Integral constraints are imposed on the players' controls, and the pursuers use quasi-strategies. The goal of the group of pursuers is to bring at least one of them within any predetermined distance of the evader. It is proven that if the total energy of the pursuers is greater than the energy of the evader, then the capture occurs in the game.

Keywords: differential games, pursuer, evader, capture, fractional derivative.

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Introduction

The modern theory of differential games is developing to a greater extent as a theory of the control of dynamical systems with geometric constraints on the players' controls. But conflict-controlled processes with integral constraints on controls are of no less importance for applications. The use of methods developed for analysis of games with geometric constraints to investigate games with integral constraints is a challenging problem. For example, the approach to describing the structure of a game that is based on operator constructions [1] is not extended immediately to games with integral constraints. The first method of L. S. Pontryagin [2] for the case of integral constraints in two-player differential games was developed in [3, 4]. The above-mentioned problem using the technique of D. Zonnevend [5] was studied in [6]. Studies using similar methods and concerned with the pursuit problem with integral constraints are presented in [7, 8].

The extremal aiming rule of N. N. Krasovskii [9] for the case of integral constraints on the players' controls was applied in [10, 11]. The method of resolving functions in the two-player game with integral constraints on the players' controls was considered in [12]. The problem of pursuit in the two-player game with generalized integral constraints on the players' controls was discussed in [13, 14]. The rule of parallel pursuit in the problem of simple pursuit with integral constraints on the controls was considered in [15]. Conditions for solvability of the problems of pursuit-evasion games of inertial players with integral constraints on the players' controls were obtained in [16]. Conditions for l-capture in a two-player game with integral constraints in which the motion of the pursuer is simple and the evader is an inertial object were obtained in [17]. The problem of pursuit in a two-player differential game described by a system of linear differential equations with a retarded argument with integral constraints was addressed in [18–20]. Two-player differential games with integral constraints on the players' controls in Hilbert space were examined in [21]. Sufficient conditions for solvability of the problem of pursuit in a game involving one evader and a group of pursuers under integral constraints on the players' controls were obtained in [22–24]. Sufficient conditions for evasion of the evader in the problem of group pursuit with integral constraints were obtained in [25]. In [26], the problem of conflict interaction between a group of pursuers and a group of evaders was treated.

This paper addresses the problem of simple group pursuit in differential games described by equations with Caputo fractional derivatives. It should be noted that, in recent years, differential games with fractional derivatives and geometric constraints on the players' controls have attracted the attention of researchers [27–29]. In this paper, sufficient conditions for solvability of the problem of simple pursuit with Caputo fractional derivatives are obtained under the condition that integral constraints are imposed on the players' controls.

§ 1. Formulation of the problem

In the space $\mathbb{R}^k$ $(k \geqslant 2)$ we consider a differential game $G(n+1)$ involving $n+1$ players: n pursuers $P_1, \dots, P_n$ and one evader E.

The law of motion of each of the pursuers P_i has the form

$$(D^p)x_i = u_i, \quad x_i(0) = x_i^0, \quad u_i \in U_i. \tag{1.1}$$

The law of motion of the evader E has the form

$$(D^p)y = v, \quad y(0) = y^0, \quad v \in V. \tag{1.2}$$

Here $i \in I = \{1, \dots, n\}$, x_i, y, x_i^0, $y^0 \in \mathbb{R}^k$, $(D^p)z$ is the Caputo derivative of order $p \in (0,1)$ of the function z,

$$\begin{aligned} U_i &= \Big\{u_i(\cdot) \mid u_i(\cdot) \ \text{ is a measurable function, } \int_0^{+\infty} \|u_i(s)\|^2\, ds \leqslant \alpha_i^2\Big\}, \\ V &= \big\{v(\cdot) \mid v(\cdot) \ \text{ is a measurable function, } \int_0^{+\infty} \|v(s)\|^2\, ds \leqslant \beta^2\big\}. \end{aligned} \tag{1.3}$$

The norm in (1.3) is Euclidean and α_i, $i \in I$, β are positive real numbers.

The function $w(\cdot) \in V$, $w \in U_i$, $i \in I$, is called *admissible*, if the function

$$t \to \int_0^t (t-s)^{p-1} w(s)\, ds$$

is defined on $[0, +\infty)$. The prehistory $v_t(\cdot)$ of the function $v(\cdot)$ at time t is the restriction of the function $v(\cdot)$ to $[0, t]$. Denote

$$\|z\|_2 = \left(\int_0^{+\infty} z^2(s)\, ds\right)^{1/2}, \quad z_i^0 = x_i^0 - y^0.$$

Definition 1.1. The quasi-strategy $\mathcal{U}_i$ of a pursuer P_i is a map $\widetilde{U}_i$ that associates a measurable function $u_i\colon [0, \infty) \to \mathbb{R}^2$ such that $\|u\|_2 \leqslant \alpha_i$ to the initial positions x_i^0, y^0 and to the prehistory $v_t(\cdot)$ of the control $v(\cdot)$ of the evader E.

Definition 1.2. A capture occurs in the game $G(n+1)$ if for any $\varepsilon > 0$ there exist time $T > 0$ and quasi-strategies $\mathcal{U}_1, \dots, \mathcal{U}_n$ of the pursuers $P_1, \dots, P_n$ such that for any admissible function $v(\cdot)$ there are time $\tau \in [0, T]$ and a number $l \in I$ for which

$$\|x_l(\tau) - y(\tau)\| < \varepsilon.$$

In what follows we will assume that all functions $v(\cdot)$ are defined on $[0, +\infty)$ and that $v(s) = 0$ for all $s \geqslant \tau$ if the equation $\displaystyle\int_0^{\tau} \|v(s)\|^2\, ds = \beta^2$ is satisfied at time τ.

§ 2. Auxiliary statements

Lemma 2.1. *Let* $p \in (0,1)$, $p \neq \frac{1}{2}$, $A, B \in (0,+\infty)$, $T_0 \geqslant 0$. *Then, there exist* $T > T_0$ *and a function* f *such that*

$$\int_{T_0}^{T} f^2(s)\,ds = A, \quad \int_{T_0}^{T} (T-s)^{p-1} f(s)\,ds = B.$$

P r o o f. We will search for the function f in the form $f(s) = K$ for all $s \in [T_0, T]$, where K is a positive constant. We obtain the system of equations

$$K^2(T-T_0) = A, \quad K(T-T_0)^p = B_1, \quad \text{where} \quad B_1 = pB.$$

Hence,

$$K^2(T-T_0) = A, \quad K^2(T-T_0)^{2p} = B_1^2.$$

Since $p \neq \frac{1}{2}$, we obtain by dividing the second equation by the first:

$$(T-T_0)^{2p-1} = C, \quad \text{where} \quad C = B_1^2/A.$$

Hence, $T = T_0 + C^{\frac{1}{2p-1}}$. From T, we find K. This proves the lemma. □

Lemma 2.2. *Let* $A, B \in (0,+\infty)$, $B^2 > 4A$, $T_0 \geqslant 0$. *Then, there exist* $T > T_0$ *and a function* f *such that*

$$\int_{T_0}^{T} f^2(s)\,ds = A, \quad \int_{T_0}^{T} \frac{f(s)}{\sqrt{T-s}}\,ds = B.$$

P r o o f. We will search for the function f in the form $f(s) = \dfrac{K}{\sqrt{T-s+2}}$, $s \in [T_0, T]$, where $K > 0$. We obtain the system of equations

$$K^2 \ln\frac{T-T_0+2}{2} = A, \quad K \ln\bigl(T-T_0+1+\sqrt{(T-T_0+1)^2-1}\,\bigr) = B.$$

Denote $t = T - T_0 + 1$. We note that $t \geqslant 1$. Taking the square of the second equation and dividing by the first, we obtain

$$g(t) = \frac{\ln^2\bigl(t+\sqrt{t^2-1}\,\bigr)}{\ln\bigl((t+1)/2\bigr)} = \frac{B^2}{A}. \tag{2.1}$$

Since $\lim\limits_{t\to 1+} g(t) = 4$, $\lim\limits_{t\to+\infty} g(t) = +\infty$, by virtue of the condition of the lemma there exists $t \geqslant 1$, the root of equation (2.1). From T, we find K. This proves the lemma. □

Lemma 2.3. *Let* $p \in (0,1)$, $0 \leqslant T_1 < T_2$. *Then*

$$\lim_{t\to+\infty} \int_{T_1}^{T_2} (t-s)^{2p-2}\,ds = 0.$$

P r o o f. The validity of the statement of the lemma is proved by straightforward verification. □

§ 3. Sufficient conditions for solvability of the pursuit problem

Theorem 3.1. *Let $n = 1$, $\alpha_1 > \beta$. Then, a capture occurs in the game $G(2)$.*

P r o o f. Let $v(\cdot)$ be an arbitrary admissible control of the evader E. Define the control of the pursuer P_1 as follows:

$$u_1(t) = v(t) - \frac{z_1^0}{\|z_1^0\|} f(t), \quad t \in [0, T].$$

The function f and time T will be defined below. Then, from the systems (1.1) and (1.2), we have

$$x_1(t) - y(t) = z_1^0 + \frac{1}{\Gamma(p)} \int_0^t (t-s)^{p-1}(u_1(s) - v(s))\, ds$$
$$= \frac{z_1^0}{\|z_1^0\|\Gamma(p)} \left(\|z_1^0\|\Gamma(p) - \int_0^t (t-s)^{p-1} f(s)\, ds \right).$$

Here $\Gamma(\cdot)$ is a gamma function. Let $\delta > 0$ be such that $\delta \leqslant \alpha_1 - \beta$, $\|z_1^0\|\Gamma(p) > 2\delta$. We choose the number T and the function f so that the following equations are satisfied:

$$\int_0^T f^2(s)\, ds = \delta^2, \quad \|z_1^0\|\Gamma(p) = \int_0^T (T-s)^{p-1} f(s)\, ds.$$

By virtue of Lemmas 2.1 and 2.2, a solution to this system exists. Assume that $f(s) = 0$ for all $s > T$. Therefore,

$$\|u_1\|_2 \leqslant \|v\|_2 + \|f\|_2 \leqslant \beta + \alpha_1 - \beta = \alpha_1.$$

Hence, the control $u_1(\cdot)$ is admissible and $x_1(T) - y(T) = 0$. This implies that a capture occurs in the game $G(2)$. This proves the theorem. □

Theorem 3.2. *Let $n = 2$ and $\alpha_1^2 + \alpha_2^2 > \beta^2$. Then, a capture occurs in the game $G(3)$.*

P r o o f. If $\alpha_1 > \beta$ or $\alpha_2 > \beta$, then the capture follows from Theorem 3.1. We assume that $\alpha_1 \leqslant \beta$, $\alpha_2 \leqslant \beta$. Let ε be an arbitrary positive number and $v(\cdot)$ be an admissible control of the evader E. Take $\delta > 0$ for which the following inequalities are satisfied: $\alpha_i > \delta$, $\|z_i^0\|\Gamma(p) > 2\delta$, $i \in I$, $(\alpha_1 - \delta)^2 + (\alpha_2 - \delta)^2 > \beta^2$. Next, let time T_1 and the function f_1 be a solution to the system

$$\int_0^{T_1} f_1^2(s)\, ds = \delta^2, \quad \int_0^{T_1} (T_1 - s)^{p-1} f_1(s)\, ds = \|z_1^0\|\Gamma(p).$$

By virtue of Lemmas 2.1 and 2.2, a solution to this system exists. Define the function

$$h_1(t) = \int_0^t \|v(s)\|^2\, ds, \quad t \geqslant 0.$$

Define the controls of the pursuers P_1 and P_2 as follows. If $h_1(t) \leqslant (\alpha_1 - \delta)^2$ and $t \in [0, T_1]$, then we set

$$u_1(t) = v(t) - \frac{z_1^0}{\|z_1^0\|} f_1(t), \quad u_2(t) = 0.$$

We show that, if $h_1(T_1) \leqslant (\alpha_1 - \delta)^2$, then a capture occurs in the game $G(3)$. From the systems (1.1) and (1.2), we have

$$x_1(T_1) - y(T_1) = z_1^0 + \frac{1}{\Gamma(p)} \int_0^{T_1} (T_1 - s)^{p-1}(u_1(s) - v(s))\, ds$$
$$= \frac{z_1^0}{\Gamma(p)\|z_1^0\|} \left(\|z_1^0\|\Gamma(p) - \int_0^{T_1} (T_1 - s)^{p-1} f_1(s)\, ds \right) = 0.$$

Assuming that $u_1(t) = 0$, $f_1(t) = 0$, $v(t) = 0$ for all $t > T_1$, we obtain

$$\|u_1\|_2 \leqslant \|v\|_2 + \|f_1\|_2 \leqslant \alpha_1 - \delta + \delta = \alpha_1.$$

Let there exist an instant $\tau \in [0, T_1)$ for which $h_1(\tau) = (\alpha_1 - \delta)^2$. We assume $u_1(t) = 0$ for all $t > \tau$. Choose time $T_1^0 > \tau$ so that the inequality

$$\left\| \frac{1}{\Gamma(p)} \int_0^\tau (t-s)^{p-1} w(s)\, ds \right\| < \varepsilon$$

is satisfied for all $t > T_1^0$ and any admissible function $w(\cdot)$.

We note that such a time T_1^0 exists since, according to the Cauchy–Bunyakovskii inequality, we have

$$\begin{aligned}\left\| \frac{1}{\Gamma(p)} \int_0^\tau (t-s)^{p-1} w(s)\, ds \right\| &\leqslant \frac{1}{\Gamma(p)} \left(\int_0^\tau (t-s)^{2p-2}\, ds \right)^{1/2} \cdot \left(\int_0^\tau \|w(s)\|^2\, ds \right)^{1/2} \\ &\leqslant \frac{\beta}{\Gamma(p)} \left(\int_0^\tau (t-s)^{2p-2}\, ds \right)^{1/2}.\end{aligned}$$

It remains to apply Lemma 2.3. Define the control of pursuer P_2 on $[\tau, T_1^0]$, assuming $u_2(t) = v(t)$ for all $t \in [\tau, T_1^0]$.

By virtue of Lemmas 2.1 and 2.2, there exist time $T_2 > T_1^0$ and a function f_2 for which the following equations hold:

$$\|z_2^0\|\Gamma(p) = \int_{T_1^0}^{T_2} (T_2 - s)^{p-1} f_2(s)\, ds, \qquad \int_{T_1^0}^{T_2} f_2^2(s)\, ds = \delta^2.$$

We take the control for pursuer P_2 to be equal to

$$u_2(t) = \begin{cases} v(t) - \dfrac{z_2^0}{\|z_2^0\|} f_2(t), & t \in [T_1^0, T_2], \\ 0 & t > T_2. \end{cases}$$

We show that the function $u_2(\cdot)$ satisfies the inequality $\|u_2\|_2 \leqslant \alpha_2$. Denote

$$\widehat{f}_2(t) = \begin{cases} 0, & t \notin [T_1^0, T_2], \\ \dfrac{z_2^0}{\|z_2^0\|} f_2(t), & t \in [T_1^0, T_2], \end{cases} \qquad \widehat{v}(t) = \begin{cases} 0, & t \notin [\tau, T_2], \\ v(t), & t \in [\tau, T_2]. \end{cases}$$

Then $u_2(t) = \widehat{v}(t) - \widehat{f}_2(t)$. Therefore, $\|u_2\|_2 \leqslant \|\widehat{v}\|_2 + \|\widehat{f}_2\|_2$. Since

$$\|\widehat{v}\|_2^2 = \int_0^{+\infty} \|\widehat{v}(s)\|^2\, ds = \int_\tau^{T_2} \|v(s)\|^2\, ds \leqslant \int_0^{+\infty} \|v(s)\|^2\, ds - \int_0^\tau \|v(s)\|^2\, ds \leqslant \beta^2 - (\alpha_1 - \delta)^2,$$

$$\|\widehat{f}_2\|_2^2 = \int_0^{+\infty} \|\hat{f}_2(s)\|^2\, ds = \int_{T_1^0}^{T_2} \|f_2(s)\|^2\, ds = \delta^2,$$

it follows that $\|u_2\|_2 \leqslant \sqrt{\beta^2 - (\alpha_1 - \delta)^2} + \delta \leqslant \alpha_2$ by virtue of the choice of δ. Therefore, the control $u_2(\cdot)$ is admissible. Next, we have

$$\begin{aligned} x_2(T_2) - y(T_2) &= z_2^0 - \int_0^\tau (T_2 - s)^{p-1} v(s)\, ds + \int_{T_1^0}^{T_2} (T_2 - s)^{p-1} (u_2(s) - v(s))\, ds \\ &= \frac{z_2^0}{\|z_2^0\|\Gamma(p)} \left(\|z_2^0\|\Gamma(p) - \int_{T_1^0}^{T_2} (T_2 - s)^{p-1} f_2(s)\, ds \right) - \int_0^\tau (T_2 - s)^{p-1} v(s)\, ds. \end{aligned}$$

Therefore, $\|x_2(T_2) - y(T_2)\| < \varepsilon$. This implies that a capture occurs in the game $G(3)$. This proves the theorem. □

Theorem 3.3. *Let $n \geqslant 3$ and $\alpha_1^2 + \ldots + \alpha_n^2 > \beta^2$. Then, a capture occurs in the game $G(n+1)$.*

P r o o f. If there exists $I_0 \subset I$, $|I_0| \leqslant n-1$, for which $\sum_{i \in I_0} \alpha_i^2 > \beta^2$, then we will consider a game in which only pursuers with the numbers from I_0 participate. Therefore, we will assume that for any $I_0 \subset I$, $|I_0| \leqslant n-1$, the inequality $\sum_{i \in I_0} \alpha_i^2 \leqslant \beta^2$ is satisfied.

Let ε be an arbitrary positive number, let $\delta > 0$ be such that $\alpha_i > \delta$, $\|z_i^0\|\Gamma(p) > 2\delta$ for all $i \in I$ and $\sum_{i \in I}(\alpha_i - \delta)^2 > \beta^2$, and let $v(\cdot)$ be an arbitrary control of the evader E.

1. Let time $T_1 > 0$ and the function f_1 satisfy the system

$$\int_0^{T_1} f_1^2(s)\, ds = \delta^2, \qquad \int_0^{T_1} (T_1 - s)^{p-1} f_1(s)\, ds = \|z_1^0\|\Gamma(p).$$

By virtue of Lemmas 2.1 and 2.2, such T_1 and f_1 exist. Consider the function

$$h_1(t) = \int_0^t \|v(s)\|^2\, ds.$$

Define the controls of the pursuers $P_i, i \in I$, as follows.

For all $t \in [0, T_1]$ for which $h_1(t) \leqslant (\alpha_1 - \delta)^2$ we assume

$$u_1(t) = v(t) - \frac{z_1^0}{\|z_1^0\|} f_1(t), \quad u_2(t) = \ldots = u_n(t) = 0.$$

Two cases are possible.

1a. $h_1(T_1) \leqslant (\alpha_1 - \delta)^2$. Then, from Theorem 3.2, it follows that a capture occurs in the game $G(n+1)$.

1b. At some time $\tau_1 < T_1$ the equation $h_1(\tau_1) = (\alpha_1 - \delta)^2$ is satisfied. We assume that $u_1(t) = 0$ for all $t > \tau_1$. The controls of the other pursuers for $t > \tau_1$ will be defined below. Let T_1^0 be a time instant for which

$$\left\| \frac{1}{\Gamma(p)} \int_0^{\tau_1} (t - s)^{p-1} w(s)\, ds \right\| < \varepsilon$$

for all $t > T_1^0$ and any admissible control $w(\cdot)$ of the evader E. In Theorem 3.2 it is shown that such a time instant T_1^0 exists.

2. Define time $T_2 > T_1^0$ and the function f_2 as a solution of the system

$$\int_{T_1^0}^{T_2} f_2^2(s)\, ds = \delta^2, \qquad \int_{T_1^0}^{T_2} (T_2 - s)^{p-1} f_2(s)\, ds = \|z_2^0\|\Gamma(p).$$

By virtue of Lemmas 2.1 and 2.2, such T_2 and f_2 exist. Consider the function

$$h_2(t) = \int_{\tau_1}^t \|v(s)\|^2\, ds.$$

Define the controls of pursuers P_i, $i \in I \setminus \{1\}$, as follows. If $h_2(t) \leqslant (\alpha_2 - \delta)^2$ and $t \in [\tau_1, T_1^0]$ we assume

$$u_2(t) = v(t), \quad u_3(t) = \ldots = u_n(t) = 0.$$

If $h_2(t) \leqslant (\alpha_2 - \delta)^2$ and $t \in [T_1^0, T_2]$, we assume

$$u_2(t) = v(t) - \frac{z_2^0}{\|z_2^0\|} f_2(t), \quad u_3(t) = \ldots = u_n(t) = 0.$$

Two cases are possible.

2a. $h_2(T_2) \leqslant (\alpha_2 - \delta)^2$. We show that in this case a capture occurs in the game $G(n+1)$. Assuming that $u_2(t) = 0$ for all $t > T_2$, we prove that the function $u_2(\cdot)$ is admissible. Denote

$$\widehat{f}_2(t) = \begin{cases} 0, & t \notin [T_1^0, T_2], \\ \dfrac{z_2^0}{\|z_2^0\|} f_2(t), & t \in [T_1^0, T_2], \end{cases} \qquad \widehat{v}(t) = \begin{cases} 0, & 0 \notin [\tau_1, T_2], \\ v(t), & t \in [\tau_1, T_2]. \end{cases}$$

Then $u_2(t) = \widehat{v}(t) - \widehat{f}_2(t)$ for all $t \geqslant 0$. This gives

$$\|u_2\|_2 \leqslant \|\widehat{v}\|_2 + \|f_2\|_2 \leqslant \alpha_2 - \delta + \delta = \alpha_2.$$

In addition,

$$\begin{aligned} x_2(T_2) - y(T_2) &= z_2^0 + \frac{1}{\Gamma(p)} \int_0^{T_2} (T_2 - s)^{p-1} (u_2(s) - v(s))\, ds \\ &= z_2^0 + \frac{1}{\Gamma(p)} \int_0^{\tau_1} (T_2 - s)^{p-1} (-v(s))\, ds + \frac{1}{\Gamma(p)} \int_{T_1^0}^{T_2} (T_2 - s)^{p-1} (u_2(s) - v(s))\, ds \\ &= \frac{z_2^0}{\|z_2^0\| \Gamma(p)} \left(\|z_2^0\| \Gamma(p) - \int_{T_1^0}^{T_2} (T_2 - s)^{p-1} f_2(s)\, ds \right) - \frac{1}{\Gamma(p)} \int_0^{\tau_1} (T_2 - s)^{p-1} v(s)\, ds. \end{aligned}$$

It follows from the definition of T_1^0 and T_2 that

$$\|x_2(T_2) - y(T_2)\| < \varepsilon,$$

which implies that a capture occurs in the game $G(n+1)$.

2b. There exists time $\tau_2 \in [\tau_1, T_2)$ for which $h_2(\tau_2) = (\alpha_2 - \delta)^2$. In this case we assume $u_2(t) = 0$ for all $t > \tau_2$, and the controls of the other pursuers will be defined below.

3. Assume that the control is constructed for the pursuer P_{l-1}, $l < n$, by the algorithm for constructing the control for the pursuer P_2. Let τ_{l-1} be a time instant for which $h_{l-1}(\tau_{l-1}) = (\alpha_{l-1} - \delta)^2$. Note that $u_l(t) = 0$ for all $t \leqslant \tau_{l-1}$. Let $T_{l-1}^0 > \tau_{l-1}$ be a time instant such that

$$\left\| \int_0^{\tau_{l-1}} (t - s)^{p-1} w(s)\, ds \right\| < \varepsilon$$

for all $t > T_{l-1}^0$ and any admissible control $w(\cdot)$ of the evader E. Define time $T_l > T_{l-1}^0$ and the function f_l as a solution of the system

$$\int_{T_{l-1}^0}^{T_l} f_l^2(s)\, ds = \delta^2, \qquad \int_{T_{l-1}^0}^{T_l} (T_l - s)^{p-1} f_l(s)\, ds = \|z_l^0\| \Gamma(p).$$

Consider the function

$$h_l(t) = \int_{\tau_{l-1}}^{t} \|v(s)\|^2\, ds.$$

If $h_l(t) \leqslant (\alpha_l - \delta)^2$ and $t \in [\tau_{l-1}, T_{l-1}^0]$, then we assume

$$u_l(t) = v(t), \quad u_{l+1}(t) = \ldots = u_n(t) = 0.$$

If $h_l(t) \leqslant (\alpha_l - \delta)^2$ and $t \in [T_{l-1}^0, T_l]$, then we assume

$$u_l(t) = v(t) - \frac{z_l^0}{\|z_l^0\|} f_l(t), \quad u_{l+1}(t) = \ldots = u_n(t) = 0.$$

The following cases are possible.

3a. $h_l(T_l) \leqslant (\alpha_l - \delta)^2$. We show that in this case a capture occurs in the game $G(n+1)$. Setting $u_l(t) = 0$ for all $t > T_l$, we prove that the function $u_l(\cdot)$ is an admissible control for pursuer P_l. Denote

$$\widehat{f}_l(t) = \begin{cases} 0, & t \notin [T^0_{l-1}, T_l], \\ \dfrac{z^0_l}{\|z^0_l\|} f_l(t), & t \in [T^0_{l-1}, T_l], \end{cases} \qquad \widehat{v}(t) = \begin{cases} 0, & 0 \notin [\tau_{l-1}, T_l], \\ v(t), & t \in [\tau_{l-1}, T_l]. \end{cases}$$

Then $u_l(t) = \widehat{v}(t) - \widehat{f}_l(t)$ for all $t \geqslant 0$, whence

$$\|u_l\|_2 \leqslant \|\widehat{v}\|_2 + \|f_l\|_2 \leqslant \alpha_l - \delta + \delta = \alpha_l.$$

In addition,

$$\begin{aligned} x_l(T_l) - y(T_l) &= z^0_l + \frac{1}{\Gamma(p)} \int_0^{T_l} (T_l - s)^{p-1}(u_l(s) - v(s))\, ds = \\ &= z^0_l + \frac{1}{\Gamma(p)} \int_0^{\tau_{l-1}} (T_l - s)^{p-1}(-v(s))\, ds + \frac{1}{\Gamma(p)} \int_{T^0_{l-1}}^{T_l} (T_l - s)^{p-1}(u_l(s) - v(s))\, ds = \\ &= \frac{z^0_l}{\|z^0_l\|\Gamma(p)} \left(\|z^0_l\|\Gamma(p) - \int_{T^0_{l-1}}^{T_l} (T_l - s)^{p-1} f_l(s)\, ds \right) - \frac{1}{\Gamma(p)} \int_0^{\tau_{l-1}} (T_l - s)^{p-1} v(s)\, ds. \end{aligned}$$

It follows from the definition of T^0_{l-1} and T_l that

$$\|x_l(T_l) - y(T_l)\| < \varepsilon,$$

which implies that a capture occurs in the game $G(n+1)$.

3b. There exists time $\tau_l \in [\tau_{l-1}, T_l)$ for which $h_l(\tau_l) = (\alpha_l - \delta)^2$. In this case we assume $u_l(t) = 0$ for all $t > \tau_l$, and if $l + 1 < n$, then we define the control u_{l+1} for the pursuer P_{l+1} similarly to the way it was done for P_l.

4. Let $l + 1 = n$. We note that then

$$\int_0^{\tau_{n-1}} \|v(s)\|\, ds = (\alpha_1 - \delta)^2 + \ldots + (\alpha_{n-1} - \delta)^2$$

and $u_n(t) = 0$ for all $t \in [0, \tau_{n-1}]$. Let $T^0_n > \tau_{n-1}$ be a time instant for which for all $t > T^0_n$ the inequality

$$\left\| \frac{1}{\Gamma(p)} \int_0^{\tau_{n-1}} (t - s)^{p-1} w(s)\, ds \right\| < \varepsilon$$

holds for all $t > T^0_{n-1}$ and any admissible function $w(\cdot)$ of the evader E. Define time $T_n > T^0_{n-1}$ and the function f_n as a solution of the system

$$\int_{T^0_{n-1}}^{T_n} f^2_n(s)\, ds = \delta^2, \qquad \int_{T^0_{n-1}}^{T_n} (T_n - s)^{p-1} f_n(s)\, ds = \|z^0_n\|\Gamma(p).$$

Let

$$h_n(t) = \int_{\tau_{n-1}}^{t} \|v(s)\|^2\, ds.$$

Then

$$h_n(T_n) \leqslant \int_0^{+\infty} \|v(s)\|^2\, ds - \int_0^{\tau_{n-1}} \|v(s)\|^2\, ds \leqslant \beta^2 - (\alpha_1 - \delta)^2 - \ldots - (\alpha_{n-1} - \delta)^2 \leqslant (\alpha_n - \delta)^2,$$

by virtue of the choice of δ. Define the control $u_n(\cdot)$ of the pursuer P_n as follows:

$$u_n(t) = \begin{cases} v(t), & t \in [\tau_{n-1}, T^0_{n-1}], \\ v(t) - \dfrac{z^0_n}{\|z^0_n\|} f_n(t), & t \in [T^0_{n-1}, T_n], \\ 0, & t > T_n. \end{cases}$$

Let us show that the function $u_n(\cdot)$ is admissible. Denote

$$\widehat{f}_n(t) = \begin{cases} 0, & t \notin [T^0_{n-1}, T_n], \\ \dfrac{z^0_n}{\|z^0_n\|} f_n(t), & t \in [T^0_{n-1}, T_n], \end{cases} \qquad \widehat{v}(t) = \begin{cases} 0, & 0 \notin [\tau_{n-1}, T_n], \\ v(t), & t \in [\tau_{n-1}, T_n]. \end{cases}$$

Then

$$\int_0^{+\infty} \|\widehat{v}(s)\|^2\, ds = \int_{\tau^0_{n-1}}^{T_n} \|v(s)\|^2\, ds \leqslant \int_0^{+\infty} \|v(s)\|^2\, ds - \int_0^{\tau_{n-1}} \|v(s)\|^2\, ds \leqslant$$
$$\leqslant \beta^2 - (\alpha_1 - \delta)^2 - \ldots - (\alpha_{n-1} - \delta)^2,$$
$$\int_0^{+\infty} \|\widehat{f}_n(s)\|^2\, ds = \int_{T^0_{n-1}}^{T_n} \|f_n(s)\|^2\, ds = \delta^2.$$

Since $u_n(t) = \widehat{v}(t) - \widehat{f}_n(t)$ for all $t \geqslant 0$, it follows that

$$\|u_n\|_2 \leqslant \|\widehat{v}\|_2 + \|\widehat{f}_n\|_2 \leqslant \sqrt{\beta^2 - (\alpha_1 - \delta)^2 - \ldots - (\alpha_{n-1} - \delta)^2} + \delta \leqslant \alpha_n,$$

by virtue of the choice of δ.

In addition, we will have

$$x_n(T_n) - y(T_n) =$$
$$= z^0_n - \frac{1}{\Gamma(p)} \int_0^{T^0_{n-1}} (T_n - s)^{p-1} v(s)\, ds + \frac{1}{\Gamma(p)} \int_{T^0_{n-1}}^{T_n} (T_n - s)^{p-1} (u_n(s) - v(s))\, ds$$
$$= \frac{z^0_n}{\|z^0_n\|\Gamma(p)} \left(\|z^0_n\|\Gamma(p) - \int_{T^0_{n-1}}^{T_n} (T_n - s)^{p-1} f_n(s)\, ds \right) - \frac{1}{\Gamma(p)} \int_0^{T^0_{n-1}} (T_n - s)^{p-1} v(s)\, ds.$$

Hence,

$$\|x_n(T_n) - y(T_n)\| < \varepsilon,$$

which implies that a capture occurs in the game $\Gamma(n+1)$. This proves the theorem. □

Remark 3.1. The authors of [21–23] propose a pursuit method in which the pursuers sequentially perform a capture, in such a way that each pursuer performs a capture in a specific time interval while implementing an active pursuit. This implies that in the process of pursuit only one of the pursuers comes close to the evader. In this paper, the pursuers also sequentially perform a capture of the evader. But the stage of pursuit by each of the pursuers is broken down into two steps. In the first step, the pursuer uses the same control as that used by the evader. In this step, they do not come close together. In the second step, the pursuer also implements an active pursuit.

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Nikolai Nikandrovich Petrov, Doctor of Physics and Mathematics, Professor, Main Researcher, Department of Differential Equations, Laboratory of Mathematical Control Theory, Udmurt State University, ul. Universitetskaya, 1, Izhevsk, 426034, Russia.
ORCID: https://orcid.org/0000-0002-0303-3559
E-mail: kma3@list.ru

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Н. Н. Петров

Простое групповое преследование с дробными производными и интегральными ограничениями

Ключевые слова: дифференциальная игра, преследователь, убегающий, поимка, дробная производная.

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Рассматривается задача простого преследования группой преследователей одного убегающего в дифференциальной игре, описываемой уравнениями с производными по Капуто дробного порядка из интервала $(0, 1)$. На управления игроков наложены интегральные ограничения, преследователи используют квазистратегии. Целью группы преследователей является сближение хотя бы одного из них с убегающим на любое наперед заданное расстояние. Доказано, что если суммарная энергия преследователей больше энергии убегающего, то в игре происходит поимка.

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Петров Николай Никандрович, д. ф.-м. н., профессор, главный научный сотрудник, кафедра дифференциальных уравнений, лаборатория математической теории управления, Удмуртский государственный университет, 426034, Россия, г. Ижевск, ул. Университетская, 1.
ORCID: https://orcid.org/0000-0002-0303-3559
E-mail: kma3@list.ru

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