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EVASION DIFFERENTIAL GAME OF A FASTER EVADER FROM MULTIPLE PURSUERS

We study a linear differential game involving a single evader and m pursuers, $m \geq 2$, in $\mathbb{R}^n$. The pursuers' control sets are unit balls, while the evader's control set is a ball of radius σ, where $\sigma > 1$. We say that evasion is possible if the state of the evader does not coincide with the state of any pursuer for all $t \geq 0$. To solve the evasion problem, a strategy is proposed for the evader, and it is shown that evasion is possible from any given initial positions of players. Using this strategy, we show that the maximum number of approach times of the pursuers to the evader is bounded above by $m(m+1)/2$.

Keywords: evasion linear differential game, control, evasion strategy, maneuver, evader, multiple pursuers.

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Introduction

The theory of differential games deals with conflicted situations in systems represented by differential equations. Interesting results have been obtained in the theory of multi-pursuer and single-evader differential games, which constitute a significant part of the theory of differential games. There are many works dedicated to the theory of differential games such as Isaacs [1], Pontryagin [2], Friedman [3], Hajek [4], Petrosyan [5], Pshenichnyi et al. [6], Azamov et al. [7,8], and Chikrii [9]. While several studies considered simple motion evasion differential games with many players such as Ibragimov and Salleh [10], Petrov [11], Bannikov [12], Liu et al. [13], Pan and Yuan [14], Salimi and Ferrara [15], Shiyuan and Zhihua [16], von Moll et al. [17, 18], Ye et al. [19], Ramana and Kothari [20], Fuchs et al. [21], Le Menec [22], Pashkov and Terekhov [23], Zak [24, 25].

The multiple capture of an evader in a differential game with fractional derivatives and phase restrictions is investigated in the work of Petrov [26]. Conditions have been derived on parameters and initial state that are sufficient for the trajectories of the players to meet at a certain instant of time for any counteractions of the evader.

Blagodatskikh [27] obtained the necessary and sufficient conditions for multiple capture in the group pursuit problems with equal opportunities in the presence of a group of defenders for an evader. Blagodatskikh and Petrov [28] studied a simple motion differential game of a group of pursuers and a group of evaders where all evaders use the same control.

Garcia and Bopardikar [29] considered a pursuit–evasion differential game in which a cooperative group of slow pursuers aims to capture a high-speed evader. In that paper, the notion of Cartesian ovals is used when the pursuer's capture radius is positive. Bopardikar and Suri [30] studied a k-capture problem in discrete time game and considered that k-capture occurs if at least k pursuers simultaneously reach the evader's position. They obtained a necessary and sufficient condition for the possibility of k-capture. A framework for solving pursuit–evasion games with multiple pursuers and one evader is provided by Kopparty and Ravishankar [31]. They presented two algorithms to capture the evader.

On manifolds with Euclidean metric, a simple motion differential game of many pursuers and one evader was studied by Kuchkarov et al. [32]. In the work of Kuchkarov et al. [33], a differential game of optimal approach of many pursuers and one evader was studied, and an estimate for the game's payoff functional was obtained. Also, a pursuit–evasion differential game of many pursuers and one evader with integral constraints on control functions of players was considered by Ibragimov and Kuchkarov [34], Ibragimov [35], and Ibragimov et al. [36].

In the present work, we construct a new evasion strategy for any initial positions of the players. The strategy guarantees that the evader can escape from any finite number m of pursuers, even though the evader is surrounded by the pursuers at the initial time. We introduce the concept of approach times, and show that the number of approach times doesn't exceed $m(m+1)/2$. We closely follow the work of Ibragimov [37], where evasion differential game of one evader and many slow pursuers was studied. The paper Ibragimov et al. [38] studied a linear differential game with coefficient $\lambda > 0$. In the present paper we examine the case $\lambda < 0$.

§ 1. Statement of problem

We consider an evasion linear differential game of m pursuers $x_1, \dots, x_m$ and one evader y that moves in space $\mathbb{R}^n$, $n \geq 2$. The dynamics of the game are described by the following equations

$$
\begin{aligned}
\dot{x}_i &= \lambda x_i + u_i, \quad x_i(0) = x_{i0}, \quad |u_i| \leq 1, \quad i = 1, 2, \dots, m, \\
\dot{y} &= \lambda y + v, \qquad y(0) = y_0, \qquad |v| \leq \sigma,
\end{aligned}
\tag{1.1}
$$

where x_i, x_{i0}, y, y_0, u_i, $v \in \mathbb{R}^n$, $\lambda < 0$; it is assumed that $x_{i0} \neq y_0$, $i = 1, 2, \dots, m$, $\sigma > 1$ is a given number, u_i is the control parameter of the pursuer x_i, and v is that of the evader y.

Definition 1.1. Lebesgue measurable functions $u_i(t)$, $|u_i(t)| \leq 1$, and $v(t)$, $|v(t)| \leq \sigma$, $t \geq 0$, are called controls of the pursuer x_i, $i \in \{1, 2, \dots, m\}$, and the evader y, respectively.

Let $\mathbb{U}$ and $\mathbb{V}$ be the sets of Lebesgue measurable functions $u\colon [0, +\infty) \to B_1$, and $v\colon [0, +\infty) \to B_\sigma$, respectively, where B_r is the ball in $\mathbb{R}^n$ with the radius r and center at the origin.

Definition 1.2. The operator

$$
V\big(y_0, x_{10}, \dots, x_{m0}, u_1(\cdot), \dots, u_m(\cdot)\big), \quad V\colon (\mathbb{R}^n)^{m+1} \times \mathbb{U}^m \to \mathbb{V},
$$

is called a non-anticipating strategy of the evader if, for any $\theta \geq 0$, y_0, x_{10}, $\dots$, $x_{m0} \in \mathbb{R}^n$, and for any $u_1^{(i)}(\cdot), \dots, u_m^{(i)}(\cdot) \in \mathbb{U}$, $i = 1, 2$, such that $u_1^{(1)}(t) = u_1^{(2)}(t), \dots, u_m^{(1)}(t) = u_m^{(2)}(t)$, $0 \leq t \leq \theta$, we have for all $0 \leq t \leq \theta$,

$$
V\big(y_0, x_{10}, \dots, x_{m0}, u_1^{(1)}(\cdot), \dots, u_m^{(1)}(\cdot)\big)(t) = V\big(y_0, x_{10}, \dots, x_{m0}, u_1^{(2)}(\cdot), \dots, u_m^{(2)}(\cdot)\big)(t).
$$

Definition 1.3. We say that evasion is possible in game (1.1) if there exists a strategy V of the evader such that, for any controls of pursuers, we have $x_i(t) \neq y(t)$ for all $t \geq 0$ and $i = 1, \dots, m$.

Problem 1.1. Construct a strategy V for the evader, for which evasion is possible in game (1.1).

§ 2. Evasion from one pursuer

In this section, we consider a linear evasion differential game problem of one pursuer and one evader in $\mathbb{R}^2$. The dynamics of the pursuer x and evader y are described by the equations

$$
\begin{aligned}
\dot{x} &= \lambda x + u, \quad x(0) = x_0, \\
\dot{y} &= \lambda y + v, \quad y(0) = y_0,
\end{aligned}
\tag{2.1}
$$

where x, x_0, y, y_0, u, $v \in \mathbb{R}^2$, $\lambda < 0$; $x(0) = x_0$ and $y(0) = y_0$ are the initial states of the players at $t = 0$ and suppose $x_0 \neq y_0$; the vectors u, v are the control parameters of the pursuer and evader, respectively. The controls satisfy the constraints

$$
|u(t)| \leq 1 \ \text{ and } \ |v(t)| \leq \sigma, \quad \sigma > 1, \quad t \geq 0. \tag{2.2}
$$

Of course, in this formulation, a control of the evader $v(t) = \dfrac{y_0 - x_0}{|y_0 - x_0|}\sigma$, $t \geq 0$, provides evasion. However, we would like to suggest and justify another evader's strategy, which is useful for the case of multiple pursuers.

If some controls $u(\cdot)$ and $v(\cdot)$ are chosen, the solutions of equations (2.1) are

$$x(t) = x_0 e^{\lambda t} + \int_0^t e^{\lambda(t-s)} u(s)\, ds, \quad y(t) = y_0 e^{\lambda t} + \int_0^t e^{\lambda(t-s)} v(s)\, ds.$$

Let $\xi(t) = e^{-\lambda t} x(t)$, $\eta(t) = e^{-\lambda t} y(t)$. Clearly, the equation $x(t) = y(t)$ is equivalent to the one $\xi(t) = \eta(t)$. Then, $\xi(0) = x_0$, $\eta(0) = y_0$, and

$$\xi(t) = x_0 + \int_0^t e^{-\lambda s} u(s)\, ds, \quad \eta(t) = y_0 + \int_0^t e^{-\lambda s} v(s)\, ds.$$

From now on, we refer to ξ and η as the pursuer and the evader, respectively.

We fix the numbers α and a

$$0 < \alpha < \min\left\{1, \frac{1}{2}(\sigma - 1)\right\}, \quad 0 < a < |x_0 - y_0|. \tag{2.3}$$

The pursuer ξ applies an arbitrary control $u(t) = (u_1(t), u_2(t))$, $t \geq 0$, and let $\xi(t) = (\xi_1(t), \xi_2(t))$ be the corresponding trajectory.

We now construct a strategy for the evader. First, the evader η starting from the initial time $t = 0$ moves with the constant control

$$v(t) = V_0 = (0, \sigma), \quad t \in [0, \tau), \tag{2.4}$$

i. e., $V_1(t) = 0$, $V_2(t) = \sigma$, parallel to the Oy-axis, where τ is the first time when $|\xi(t) - \eta(t)| = a$. We call τ the a-approach time of pursuer ξ to the evader η. The segment between the points y_0 and $\eta(\tau)$ in Fig. 1 is the trajectory of the evader corresponding to (2.4).

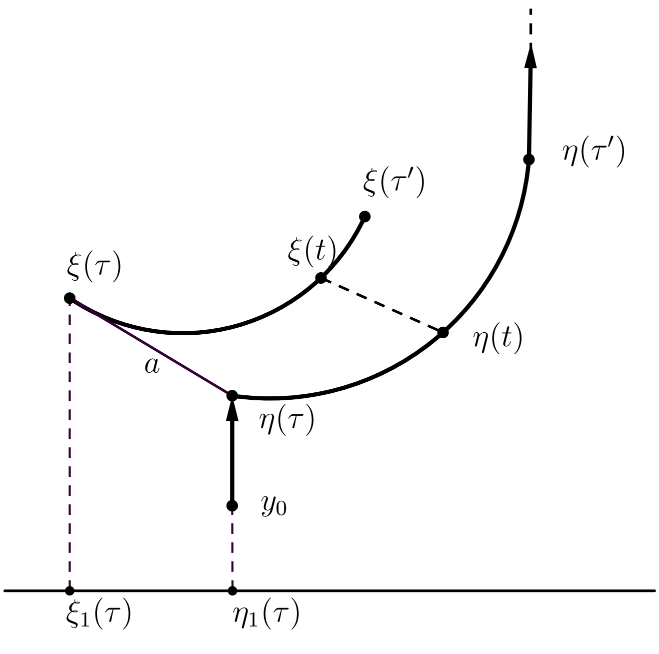

Fig. 1. The trajectory of the evader when $\xi_1(\tau) \leq \eta_1(\tau)$

Note that time τ may not occur at all. In this case, we have $|\xi(t) - \eta(t)| > a$ for all $t \geq 0$ and, clearly, $\xi(t) \neq \eta(t)$ for all $t \geq 0$. Therefore, we assume that the time τ occurs. In addition, we define the number $\tau' = -\frac{1}{\lambda} \ln\left(e^{-\lambda\tau} - \frac{2\lambda a}{\sigma-1-\alpha}\right)$. Note that the evader will apply a maneuver on the time interval $[\tau, \tau')$.

We temporarily use the notation $V(t) = (V_1(t), V_2(t))$ only in this section, where

$$V_1(t) = \begin{cases} |u_1(t)| + \alpha, & \xi_1(\tau) \leq \eta_1(\tau), \\ -(|u_1(t)| + \alpha), & \xi_1(\tau) > \eta_1(\tau), \end{cases} \qquad V_2(t) = \sqrt{\sigma^2 - V_1^2(t)}. \tag{2.5}$$

Clearly, $|V_1(t)| \leq |u_1(t)| + \frac{1}{2}(\sigma - 1) < \sigma$, and so $V_2(t)$ in (2.5) is defined. The evader applies the following strategy on $[\tau, \tau')$:

$$v(t) = (V_1(t), V_2(t)), \quad t \in [\tau, \tau'). \tag{2.6}$$

We call $V(t)$ defined by (2.6) a maneuver of the evader η against the pursuer ξ. For the final part of the evader's strategy, we let

$$V(t) = V_0 = (0, \sigma), \quad t \geq \tau'. \tag{2.7}$$

Strictly speaking, strategy (2.5)–(2.7) is not written in the non-anticipative form of Definition 1.2, since it is expressed through the players' positions at time τ. However, these positions are uniquely determined by the initial states and the pursuer's control on $[0, \tau]$. Therefore, strategy (2.5)–(2.7) is a non-anticipative strategy in the sense of Definition 1.2. An analogous observation applies to the evader's strategy in the multi-pursuer case discussed in Subsection 3.2.

The main result of this section is the following statement which will be used to prove the main result of the paper in Section 4.

Lemma 2.1. *Let the evader use strategy* (2.4), (2.6), *and* (2.7), *where τ is the a-approach time of the pursuer ξ to the evader η. Then*

$$|\eta(t) - \xi(t)| \geq a, \quad 0 \leq t \leq \tau, \tag{2.8}$$

$$|\eta(t) - \xi(t)| > \frac{\alpha a}{2\sigma}, \quad \tau \leq t \leq \tau', \tag{2.9}$$

$$\eta_2(t) - \xi_2(t) > a, \quad t \geq \tau'. \tag{2.10}$$

P r o o f. We will prove this lemma by considering the three parts of the evader's strategy defined by formulas (2.4), (2.6), (2.7), respectively. First, the evader η moves with the velocity $V(t) = (0, \sigma)$, $0 \leq t < \tau$, along a vertical line. The corresponding trajectory of the evader is a segment with the endpoints y_0 and $\eta(\tau)$ (see Fig. 1). By definition of τ we have $|\xi(t) - \eta(t)| \geq a$ for $0 \leq t \leq \tau$, and so (2.8) is true.

To prove (2.9), we consider the case $\xi_1(\tau) \leq \eta_1(\tau)$, hence, by (2.5) $V_1(t) = |u_1(t)| + \alpha$. The argument when $\xi_1(\tau) > \eta_1(\tau)$ is completely analogous. The curve between the points $\eta(\tau)$ and $\eta(\tau')$ in Fig. 1 is the trajectory of the evader corresponding to the maneuver (2.5). We have, for $\tau \leq t \leq \tau'$,

$$\begin{aligned} |\eta(t) - \xi(t)| &\geq \eta_1(t) - \xi_1(t) = \eta_1(\tau) - \xi_1(\tau) + \int_\tau^t e^{-\lambda s} V_1(s)\, ds - \int_\tau^t e^{-\lambda s} u_1(s)\, ds \\ &\geq \eta_1(\tau) - \xi_1(\tau) + \int_\tau^t e^{-\lambda s} (|u_1(s)| + \alpha)\, ds - \int_\tau^t e^{-\lambda s} |u_1(s)|\, ds \\ &\geq \alpha \int_\tau^t e^{-\lambda s} ds = \frac{\alpha}{\lambda} \left(e^{-\lambda\tau} - e^{-\lambda t}\right). \end{aligned}$$

On the other hand, in view of (2.2), we obtain

$$|\eta(t)-\xi(t)| \geq |\eta(\tau)-\xi(\tau)| - \left|\int_\tau^t e^{-\lambda s}V(s)\,ds\right| - \left|\int_\tau^t e^{-\lambda s}u(s)\,ds\right|$$
$$\geq a - \int_\tau^t e^{-\lambda s}|V(s)|\,ds - \int_\tau^t e^{-\lambda s}|u(s)|\,ds$$
$$\geq a - \frac{\sigma+1}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t}\right).$$

Hence,

$$|\eta(t)-\xi(t)| \geq f(t) = \max\left\{\frac{\alpha}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t}\right), a - \frac{\sigma+1}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t}\right)\right\}.$$

Since the function $f_1(t) = \frac{\alpha}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t}\right)$, $t \geq \tau$, is increasing, and the function $f_2(t) = a - \frac{\sigma+1}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t}\right)$, $t \geq \tau$, is decreasing, therefore the function $f(t)$, $t \geq \tau$, achieves its minimum at $t = t^*$ where $f_1(t) = f_2(t)$. We can see that $t^* = -\frac{1}{\lambda}\ln\left(e^{-\lambda\tau} - \frac{\lambda a}{\alpha+\sigma+1}\right) \in [\tau, \tau']$.

Hence, for any $t \in [\tau, \tau']$, by (2.3) we have

$$|\eta(t)-\xi(t)| \geq f(t^*) = \frac{\alpha}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t^*}\right) = \frac{\alpha a}{\sigma+1+\alpha} > \frac{\alpha a}{2\sigma},$$

which proves (2.9).

Next, to prove (2.10), we first establish that

$$\eta_2(\tau') - \xi_2(\tau') > a.$$

Indeed, for $\tau \leq t \leq \tau'$, due to the obvious inequality $\eta_2(\tau) - \xi_2(\tau) \geq -|\eta(\tau)-\xi(\tau)| = -a$ we have

$$\begin{aligned}\eta_2(t)-\xi_2(t) &= \eta_2(\tau)-\xi_2(\tau) + \int_\tau^t e^{-\lambda s}V_2(s)\,ds - \int_\tau^t e^{-\lambda s}u_2(s)\,ds \\ &\geq -a + \int_\tau^t e^{-\lambda s}\left(\sqrt{\sigma^2-(|u_1(s)|+\alpha)^2} - \sqrt{1-u_1^2(s)}\right)ds. \qquad (2.11)\end{aligned}$$

Noting that the function

$$g(\mu) = \sqrt{\sigma^2-(\mu+\alpha)^2} - \sqrt{1-\mu^2}, \quad \mu \in [0,1],$$

achieves its minimum at $\mu_0 = \frac{\alpha}{\sigma-1}$, it follows from (2.11) that

$$\eta_2(t)-\xi_2(t) \geq -a + \frac{1}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t}\right)\left(\sqrt{\sigma^2-(\mu_0+\alpha)^2} - \sqrt{1-\mu_0^2}\right)$$
$$= -a + \frac{1}{\lambda}\left(e^{-\lambda\tau}-e^{-\lambda t}\right)\sqrt{(\sigma-1)^2-\alpha^2}.$$

In particular, for $t = \tau'$, we obtain $e^{-\lambda\tau'} = e^{-\lambda\tau} - \frac{2\lambda a}{\sigma-1-\alpha}$ and so,

$$\eta_2(\tau')-\xi_2(\tau') \geq -a + \frac{2a}{\sigma-1-\alpha}\sqrt{(\sigma-1)^2-\alpha^2} > a.$$

Since $|u_2(s)| \leq 1$, and in view of (2.7) $V_2(s) = \sigma$ for $t \geq \tau'$, we have

$$\eta_2(t)-\xi_2(t) = \eta_2(\tau')-\xi_2(\tau') + \int_{\tau'}^t e^{-\lambda s}V_2(s)\,ds - \int_\tau^t e^{-\lambda s}u_2(s)\,ds$$
$$> a + \frac{1}{\lambda}\left(e^{-\lambda\tau'}-e^{-\lambda t}\right)(\sigma-1) \geq a.$$

The proof of the lemma is complete. □

In particular, Lemma 2.1 implies that even though the pursuer is on the same vertical line of the evader and above the evader, the evader can avoid from capturing using the maneuver.

§ 3. Evasion from many pursuers

We prove the following statement.

Theorem 3.1. *Let $\lambda < 0$. Then, for any initial positions of players, evasion is possible in game* (1.1).

We divide the proof into subsections.

3.1. Definitions of parameters

Let α and a_1 be any fixed numbers that satisfy the following relations

$$0 < \alpha < \min\left\{1, \frac{1}{2}(\sigma - 1)\right\}, \quad 0 < a_1 < \min\left\{\frac{\sigma - 1 - \alpha}{-4\lambda}, \min_{i=1,\dots,m} |y_0 - x_{i0}|\right\}. \tag{3.1}$$

We say that $t = \tau_1 > 0$ is the a_1-approach time of a pursuer ξ_{i_1} to the evader, if $|\xi_{i_1}(\tau_1) - \eta(\tau_1)| = a_1$ and $|\xi_i(t) - \eta(t)| > a_1$ for all $0 \le t < \tau_1$ and $i = 1, 2, \dots, m$.

Let

$$\beta = \frac{(\sigma - 1 - \alpha)\alpha}{2^6 \sigma^2 e^{-\lambda T}}. \tag{3.2}$$

where $T = \tau_1 + \frac{4a_1}{\sigma - 1 - \alpha}$. We observe

$$\beta < \frac{\sigma - 1 - \alpha}{2^6 e^{-\lambda T}}, \quad \beta < \frac{\sigma - 1 - \alpha}{2^5 \sigma e^{-\lambda T}}, \quad \beta < \frac{1}{2}. \tag{3.3}$$

We define a decreasing geometric sequence $\{a_k\}_{k=1}^{\infty}$ by the equation $a_{k+1} = \beta a_k$, $k = 1, 2, \dots$. It is easy to check that this decreasing geometric sequence has the following property:

Proposition 3.1. $\displaystyle\sum_{k=p+1}^{\infty} a_k = \frac{a_{p+1}}{1 - \beta} \le 2a_{p+1}$ *for any integer* $p \ge 0$.

Next, we assume that the evader is moving under some strategy. Let

$$\xi_i(t) = x_{i0} + \int_0^t e^{-\lambda s} u_i(s)\, ds, \quad \xi_i(t) = e^{-\lambda t} x_i(t), \quad \xi_i(0) = x_{i0}, \quad i = 1, 2, \dots, m,$$

$$\eta(t) = y_0 + \int_0^t e^{-\lambda s} v(s)\, ds, \quad \eta(t) = e^{-\lambda t} y(t), \quad \eta(0) = y_0,$$

Clearly, $x_i(t) \ne y(t)$ is equivalent to $\xi_i(t) \ne \eta(t)$.

In general, if τ_{k-1}, $k \ge 2$, is the a_{k-1}-approach time, then we define the time $t = \tau_k > \tau_{k-1}$ to be the a_k-approach time for a pursuer ξ_{i_k} if $|\xi_{i_k}(\tau_k) - \eta(\tau_k)| = a_k$ and $|\xi_i(t) - \eta(t)| > a_k$ for all $0 \le t < \tau_k$ and $i = 1, 2, \dots, m$.

Thus, we have defined a monotone increasing sequence $\tau_1 < \tau_2 < \dots$ of the approach times. Notice that the same time τ_k can be the a_k-approach time of several pursuers to the evader. Fig. 2 illustrates such a situation, where τ_k is an a_k-approach time of the pursuers ξ_i, ξ_j and ξ_k to the evader.

If there are more than one pursuers, for which τ_k is the a_k-approach time, we choose any of these pursuers and, without restriction of generality, relabel it by ξ_k. Hence, by the definition of τ_k we have

$$|\eta(t) - \xi_i(t)| > a_k, \quad i = 1, 2, \dots, m, \quad 0 \le t < \tau_k, \quad |\eta(\tau_k) - \xi_k(\tau_k)| = a_k. \tag{3.4}$$

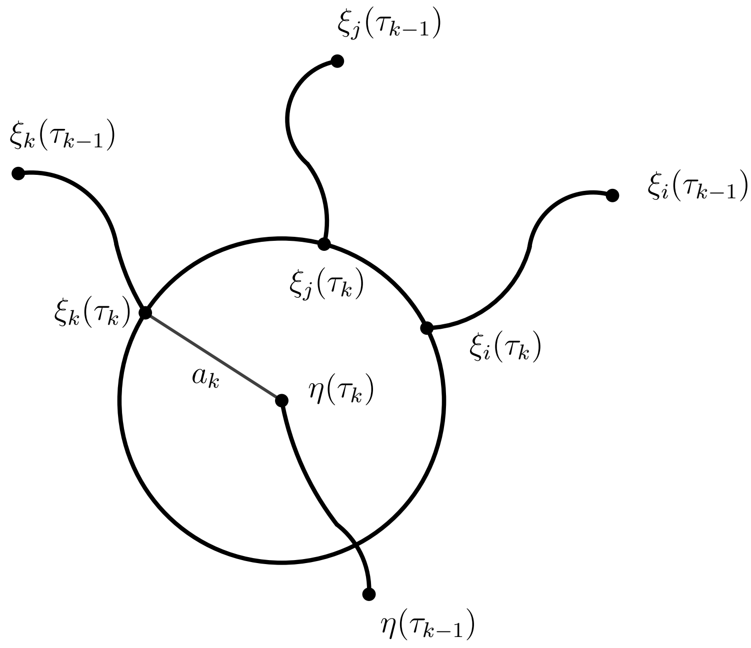

Fig. 2. τ_k is the a_k-approach time of the pursuers ξ_i, ξ_j and ξ_k to the evader η

It is worth noting that the same pursuer ξ_k can have several other approach times $\tau_{k'}$, $\tau_{k''}, \ldots,$ after the approach time τ_k.

Let

$$\tau_k' = -\frac{1}{\lambda}\ln\left(e^{-\lambda\tau_k} - \frac{2\lambda a_k}{\sigma - 1 - \alpha}\right), \quad k = 1, 2, \ldots, \quad \tau_0 = 0, \quad \tau_0' = +\infty.$$

Notice that the sequence $\tau_1',\ \tau_2',\ \tau_3', \ldots$ is not necessarily monotone increasing.

3.2. Strategy for the evader

Without loss of generality, we assume that $y_0 = (0, 0)$, that is, the evader is at the origin at the initial time. For $k = 1, 2, \ldots,$ we define the maneuvers $V_k(t) = (V_{k1}(t), V_{k2}(t))$, as follows

$$V_{k1}(t) = \begin{cases} |u_{k1}(t)| + \alpha, & \xi_{k1}(\tau_k) \le \eta_1(\tau_k), \\ -(|u_{k1}(t)| + \alpha), & \xi_{k1}(\tau_k) > \eta_1(\tau_k), \end{cases} \quad V_{k2}(t) = \sqrt{\sigma^2 - V_{k1}^2(t)}.$$

First, the evader moves starting from the time $\tau_0 = 0$ along the Oy-axis with the velocity $v(t) = V_0 = (0, \sigma)$. If the a_1-approach time $\tau_1 > 0$ doesn't occur, that is, $|\eta(t) - \xi_i(t)| > a_1$ for all $i = 1, 2, \ldots, m$ and $t \ge 0$, then, clearly, $\xi_i(t) \ne \eta(t)$, $t \ge 0$, $i = 1, 2, \ldots, m$, and so evasion is possible in the game.

Let the a_1-approach time $\tau_1 > 0$ occur. In general, the evader constructs its strategy as follows. Let the time τ_k, $k \ge 1$, occur.

(i) if the time τ_{k+1} occurs in the interval $[\tau_k, \tau_k')$, then $v(t) = V_k(t)$ on $[\tau_k, \tau_{k+1})$. (Fig. 3, *a*)

(ii) if the time τ_{k+1} occurs in $[\tau_k', \infty)$, then $v(t) = V_k(t)$ on $[\tau_k, \tau_k')$ and $v(t) = V_0$ on $[\tau_k', \tau_{k+1})$. (Fig. 3, *b*)

(iii) if the time τ_{k+1} never occurs, then $v(t) = V_k(t)$ on $[\tau_k, \tau_k')$ and $v(t) = V_0$ on $[\tau_k', \infty)$. (Fig. 3, *c*)

In other words, we construct the strategy of the evader according to the following reasoning R_k: If τ_k occurs, then the evader will start applying the maneuver V_k from time τ_k. The evader's subsequent behavior depends on the time of occurrence of τ_{k+1}. If $\tau_{k+1} \le \tau_k'$, then the

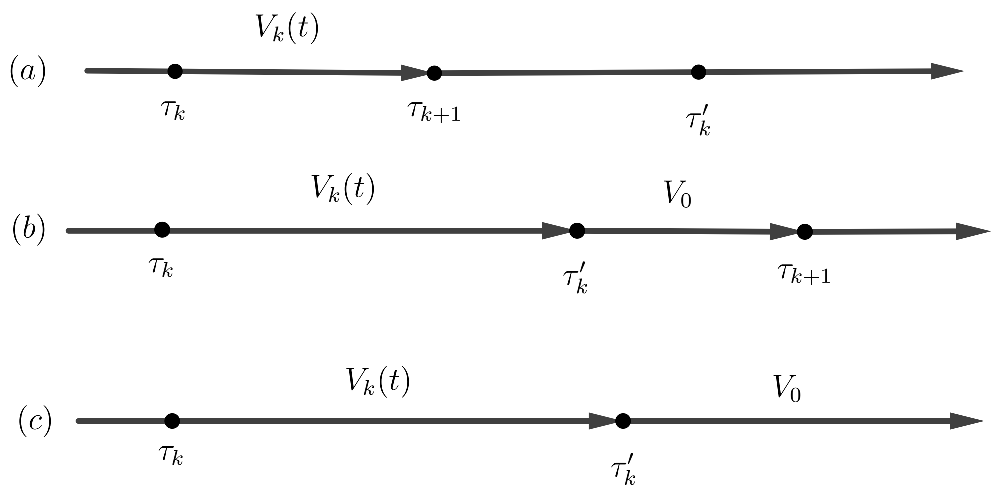

Fig. 3. The applications of $V_k(t)$ and V_0 depending on occurrence τ_{k+1}.

evader will continue applying V_k until τ_{k+1}. If $\tau_{k+1} > \tau'_k$, then the evader will apply V_k until τ'_k, and then apply V_0 until τ_{k+1}. If τ_{k+1} does not occur, the evader applies V_k until τ'_k, and then applies V_0. Starting from τ_{k+1}, the reasoning R_{k+1} is applied.

According to the description of evader's strategy (i)–(iii) there are two possible cases.

Case A. The finite approach times $\tau_1, \tau_2, \ldots, \tau_{k_1}$ with $\tau_1 < \tau_2 < \ldots < \tau_{k_1}$ occur so that $\tau_2 < \tau'_1$, $\tau_3 < \tau'_2$, $\ldots$, $\tau_{k_1} < \tau'_{k_1-1}$, and there is no an approach time in $[\tau_{k_1}, \tau'_{k_1})$ for some $k_1 \geq 1$. Then, we say that the evader is under a group attack of the pursuers ξ_1, ξ_2,$\ldots$, ξ_{k_1} on the time interval $[\tau_1, \tau'_{k_1})$. Thus, the first group attack of pursuers ends at τ'_{k_1} in Case A.

By items (i)–(iii), the strategy of the evader on the interval $[\tau_0, \tau'_{k_1})$ can be written as follows:

$$v(t) = \begin{cases} V_0, & \tau_0 \leq t < \tau_1, \\ V_k(t), & \tau_k \leq t < \tau_{k+1}, \quad k = 1, 2, \ldots, k_1 - 1, \\ V_{k_1}(t), & \tau_{k_1} \leq t < \tau'_{k_1}. \end{cases} \tag{3.5}$$

By item (ii), starting at τ'_{k_1} the evader starts to apply $v(t) = V_0$ and after some time the evader may undergo another group attack of pursuers.

Fig. 4 illustrates the three sections of the evader's trajectory between the points $\eta(\tau_1)$, $\eta(\tau_2)$, $\eta(\tau_3)$, and $\eta(\tau'_3)$ corresponding to some maneuvers $v(t) = V_1(t)$, $v(t) = V_2(t)$, and $v(t) = V_3(t)$, where $\tau_2 < \tau'_1$, $\tau_3 < \tau'_2$ and there is no an approach time in $[\tau_3, \tau'_3)$.

Since, by the definition of approach times, we have $\tau_k < \tau_{k+1}$, $k \geq 1$, therefore, in view of the conditions $\tau_{k+1} < \tau'_k$, $k = 1, \ldots, k_1 - 1$, we get in Case A the following inclusion:

$$[\tau_1, \tau'_{k_1}) = \bigcup_{k=1}^{k_1-1} [\tau_k, \tau_{k+1}) \cup [\tau_{k_1}, \tau'_{k_1}) \subset \bigcup_{k=1}^{k_1} [\tau_k, \tau'_k). \tag{3.6}$$

Case B. Let infinitely many successive approach times τ_1, τ_2, $\ldots$ of pursuers ξ_1, ξ_2, $\ldots$ to the evader occur satisfying the conditions $\tau_k < \tau_{k+1} < \tau'_k$ for all $k = 1, 2, \ldots$. Then, $[\tau_k, \tau_{k+1}) \subset [\tau_k, \tau'_k)$ and so, for any $l \geq 1$, we have

$$[\tau_1, \tau_l) = \bigcup_{k=1}^{l-1} [\tau_k, \tau_{k+1}) \subset \bigcup_{k=1}^{l-1} [\tau_k, \tau'_k), \tag{3.7}$$

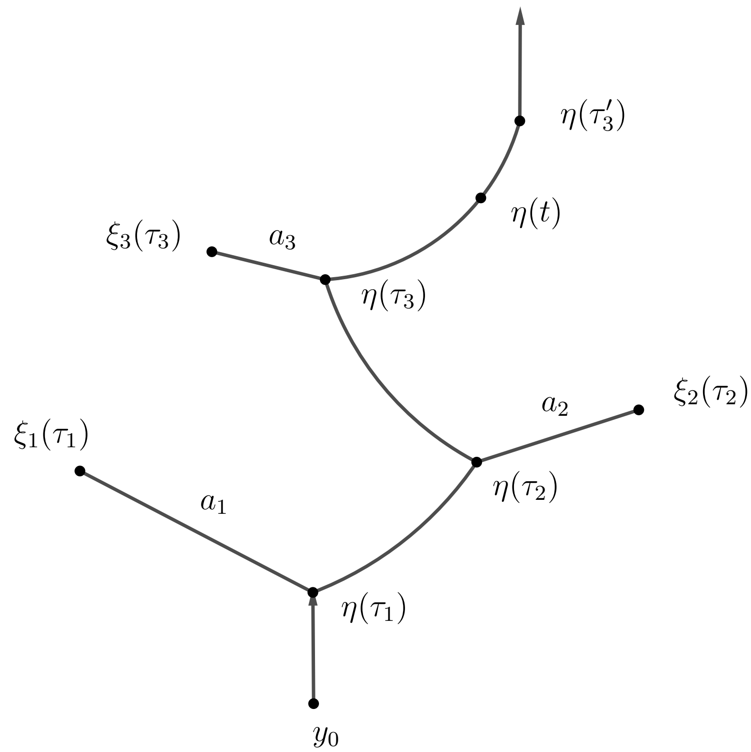

Fig. 4. The trajectory of the evader

We use the inequality $\ln(a-b) \le \ln a - b$, for any $a \ge 1$, $b \le 0$, to obtain

$$\begin{aligned} \tau_k' - \tau_k &= -\frac{1}{\lambda}\ln\left(e^{-\lambda\tau_k} - \frac{2\lambda a_k}{\sigma-1-\alpha}\right) - \tau_k \\ &\le -\frac{1}{\lambda}\left(-\lambda\tau_k - \frac{2\lambda a_k}{\sigma-1-\alpha}\right) - \tau_k = \frac{2a_k}{\sigma-1-\alpha}. \end{aligned} \tag{3.8}$$

Therefore,

$$\begin{aligned} \tau_l - \tau_1 &= \sum_{k=1}^{l-1}(\tau_{k+1}-\tau_k) < \sum_{k=1}^{l-1}(\tau_k'-\tau_k) \\ &< \sum_{k=1}^{\infty}\frac{2a_k}{\sigma-1-\alpha} = \frac{2a_1}{(\sigma-1-\alpha)(1-\beta)} < \infty. \end{aligned} \tag{3.9}$$

This means that the increasing sequence τ_l is bounded. Then, the limit $\tau_\infty = \lim_{l\to\infty}\tau_l$ exists. From (3.9), we have

$$\tau_\infty \le \tau_1 + \frac{2a_1}{(\sigma-1-\alpha)(1-\beta)} < \tau_1 + \frac{4a_1}{\sigma-1-\alpha} = T. \tag{3.10}$$

Note that in Case B inclusion (3.7) holds for any $l \ge 1$, and passing to limit as $l \to \infty$ in (3.7) we obtain

$$[\tau_1, \tau_\infty) \subseteq \bigcup_{k=1}^{\infty}[\tau_k, \tau_k'). \tag{3.11}$$

By items (i)–(iii), the evader's strategy on the interval $[\tau_1, \tau_\infty)$ (see Fig. 5) is

$$v(t) = V_k(t), \quad t \in [\tau_k, \tau_{k+1}), \quad \tau_k < \tau_{k-1}', \quad k = 1, 2, \ldots.$$

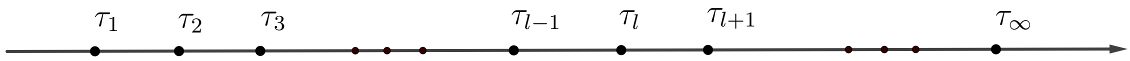

Fig. 5. The approaches

From now on we use $\bar{\tau}$ to denote τ'_{k_1} in Case A, and to denote τ_∞ in Case B. We'll discuss in detail the first group attack, which starts at the time τ_1 and ends at $\bar{\tau}$. Another group attack may occur after the time $\bar{\tau}$ as well, which can be studied in a similar fashion. We will prove that Case B is impossible.

3.3. Estimation of distance between evader and fictitious evader

Take any a_p-approach time τ_p of the pursuer ξ_p to the evader η, where $p \in \{1, 2, \ldots, k_1\}$ in Case A, and p is any positive integer in Case B, and estimate the distance between $\xi_p(t)$ and $\eta(t)$ for $t \geq \tau_p$. In order to obtain the desired estimate, we introduce for $t \in [\tau_p, \tau'_p]$ a fictitious evader (FE) ζ_p whose motion is described by the equation

$$\dot{\zeta}_p = e^{-\lambda t} w_p, \quad \zeta_p(\tau_p) = \eta(\tau_p),$$

where w_p is the control parameter of FE ζ_p. We let

$$w_p(t) = V_p(t) = (V_{p1}(t), V_{p2}(t)), \quad t \in [\tau_p, \tau'_p). \tag{3.12}$$

Then, by (2.9), we have

$$|\zeta_p(t) - \xi_p(t)| > \frac{\alpha a_p}{2\sigma}, \quad \tau_p \leq t \leq \tau'_p. \tag{3.13}$$

Moreover, by (2.10)

$$\zeta_{p2}(\tau'_p) - \xi_{p2}(\tau'_p) > a_p. \tag{3.14}$$

Note that FE ζ_p moves only on the time interval $[\tau_p, \tau'_p)$ and its initial state $\zeta(\tau_p)$ coincides with the initial state $\eta(\tau_p)$ of the real evader (see Fig. 6).

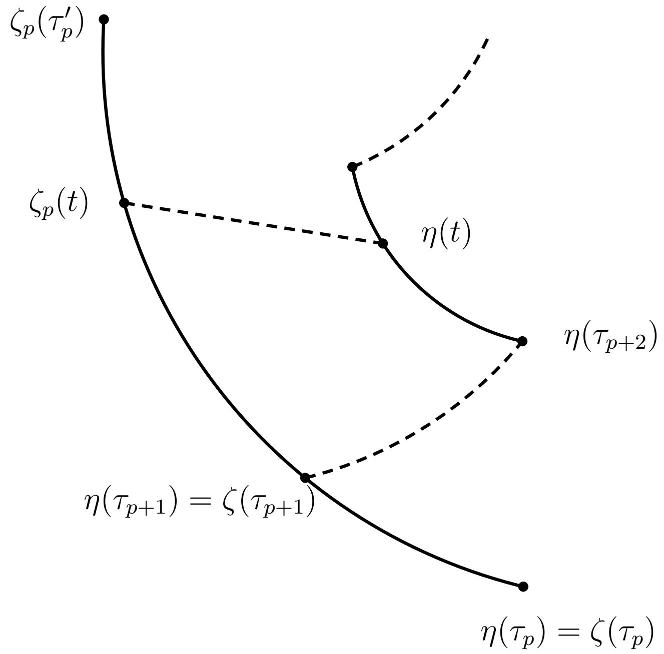

Fig. 6. Evader η and fictitious evader ζ_p.

In Case A, by (3.5), the strategy of evader on the interval $[\tau_p, \tau'_{k_1})$, $k_1 \ge p$, is

$$v(t) = \begin{cases} V_k(t), & t \in [\tau_k, \tau_{k+1}), \quad k = p, p+1, \dots, k_1 - 1, \\ V_{k1}(t), & t \in [\tau_{k1}, \tau'_{k1}), \end{cases} \tag{3.15}$$

where

$$\tau_k < \tau'_{k-1}, \quad k = p+1, p+2, \dots, k_1;$$

and in Case B, by the description (i)–(iii) the evader's strategy on the interval $[\tau_p, \tau_\infty)$ is

$$v(t) = V_k(t), \quad t \in [\tau_k, \tau_{k+1}), \quad \tau_k < \tau'_{k-1}, \quad k = p, p+1, \dots. \tag{3.16}$$

We now estimate the distance between the points $\eta(t)$ and $\zeta_p(t)$ on $[\tau_p, \bar{\tau}]$, where $\bar{\tau} = \tau'_{k_1}$ in Case A, and $\bar{\tau} = \tau_\infty$ in Case B. Since $\tau_{p+1} < \tau'_p$, we have the inclusion

$$[\tau'_p, t) \subset [\tau_{p+1}, t) \subset \bigcup_{\substack{k \ge p+1, \\ \tau_k \le t}} [\tau_k, \tau'_k), \quad \tau_{p+1} \le t \le \bar{\tau}, \tag{3.17}$$

from (3.6) and (3.11) for both Cases A and B. We recall

$$T = \tau_1 + \frac{4a_1}{\sigma - 1 - \alpha},$$

since $\tau_i < \tau'_{i-1}$, and by (3.8), and Proposition 3.1, we then have

$$\begin{aligned} \bar{\tau} &= \tau'_{k_1} = \tau_{k_1} + (\tau'_{k_1} - \tau_{k_1}) \\ &= \tau_1 + \sum_{i=2}^{k_1} (\tau_i - \tau_{i-1}) + (\tau'_{k_1} - \tau_{k_1}) \\ &< \tau_1 + \sum_{i=2}^{k_1} (\tau'_{i-1} - \tau_{i-1}) + (\tau'_{k_1} - \tau_{k_1}) \\ &= \tau_1 + \sum_{i=1}^{k_1} (\tau'_i - \tau_i) < \tau_1 + \sum_{i=1}^{\infty} (\tau'_i - \tau_i) \\ &\le \tau_1 + \sum_{i=1}^{\infty} \frac{2a_i}{\sigma - 1 - \alpha} \le \tau_1 + \frac{4a_1}{\sigma - 1 - \alpha}. \end{aligned} \tag{3.18}$$

Hence, the inequality

$$\bar{\tau} < T \tag{3.19}$$

follows from (3.18) in Case A, and from (3.10) in Case B.

Proposition 3.2. *For any $p \in \{1, \dots, k_1 - 1\}$ in Case A, and for any positive integer p in Case B, we have*

$$t - \tau_{p+1} \le \frac{4}{\sigma - 1 - \alpha} a_{p+1}, \quad t \in [\tau_{p+1}, \bar{\tau}].$$

P r o o f. By using (3.17), (3.8) and Proposition 3.1, we have

$$\begin{aligned} t - \tau_{p+1} &\le \sum_{k \ge p+1, \ \tau_k \le t} (\tau'_k - \tau_k) \le \sum_{k \ge p+1, \ \tau_k \le t} \frac{2a_k}{\sigma - 1 - \alpha} \\ &< \frac{2}{\sigma - 1 - \alpha} \sum_{k=p+1}^{\infty} a_k \le \frac{4}{\sigma - 1 - \alpha} a_{p+1}. \end{aligned}$$

The proof of the proposition is complete. □

Lemma 3.1. *Let the evader use strategy* (3.15) *in Case A and* (3.16) *in Case B. Then,*

$$|\eta(t)-\zeta_p(t)| \le \frac{16\sigma e^{-\lambda T}}{\sigma-1-\alpha}a_{p+1}, \quad t\in[\tau_p,\tau_p']\cap[\tau_p,\bar\tau]. \tag{3.20}$$

P r o o f. This lemma says that if $\tau_p' \le \bar\tau$ (see Fig. 7, *a*), then estimate (3.20) is true on $\tau_p \le t \le \tau_p'$. It should be noted that τ_p' can also be greater than $\bar\tau$ (see Fig. 7, *b*). If $\tau_p' > \bar\tau$, then (3.20) is true on $[\tau_p,\bar\tau]$.

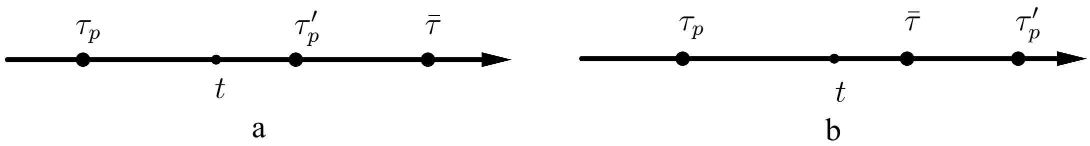

Fig. 7. The location of times τ_p' and $\bar\tau$

Note that $\eta(\tau_p)=\zeta(\tau_p)$ and if there is no an approach time in the set $[\tau_p,\tau_p']\cap[\tau_p,\bar\tau]$, then, by (3.12) and (3.15), $v(t)=w_p(t)=V_p(t)$, $t\in[\tau_p,\tau_p']\cap[\tau_p,\bar\tau]$. Therefore,

$$\eta(t)-\zeta_p(t)=\eta(\tau_p)+\int_{\tau_p}^t e^{-\lambda s}V_p(s)\,ds-\zeta_p(\tau_p)-\int_{\tau_p}^t e^{-\lambda s}V_p(s)\,ds=0, \quad t\in[\tau_p,\tau_p']\cap[\tau_p,\bar\tau],$$

and so (3.20) is satisfied.

We let now one or several approach times $\tau_{p+1}, \tau_{p+2}, \ldots$ occur in the set $[\tau_p,\tau_p']\cap[\tau_p,\bar\tau]$. In Fig. 8, the sections of the trajectory of the evader correspond to distinct maneuvers. Since by (3.12) and (3.15) $v(t)=w_p(t)=V_p(t)$, $t\in[\tau_p,\tau_{p+1}]$, we have $\eta(t)=\zeta_p(t)$, $t\in[\tau_p,\tau_{p+1}]$, therefore (3.20) is true for $t\in[\tau_p,\tau_{p+1}]$. In particular, we have $\eta(\tau_{p+1})=\zeta_p(\tau_{p+1})$.

Next, for any $t\in[\tau_{p+1},\tau_p']\cap[\tau_{p+1},\bar\tau]$, we obtain

$$\begin{aligned}
|\eta(t)-\zeta_p(t)| &= \left|\eta(\tau_{p+1})+\int_{\tau_{p+1}}^t e^{-\lambda s}v(s)\,ds-\zeta_p(\tau_{p+1})-\int_{\tau_{p+1}}^t e^{-\lambda s}V_p(s)\,ds\right| \\
&= \left|\int_{\tau_{p+1}}^t e^{-\lambda s}\big(v(s)-V_p(s)\big)\,ds\right| \le \int_{\tau_{p+1}}^t e^{-\lambda s}|v(s)-V_p(s)|\,ds \\
&\le \int_{\tau_{p+1}}^t e^{-\lambda s}\big(|v(s)|+|V_p(s)|\big)\,ds \le 2\sigma\int_{\tau_{p+1}}^t e^{-\lambda s}\,ds \\
&= -\frac{2\sigma}{\lambda}\big(e^{-\lambda t}-e^{-\lambda\tau_{p+1}}\big)= -\frac{2\sigma}{\lambda}e^{-\lambda\tau_{p+1}}\big(e^{-\lambda(t-\tau_{p+1})}-1\big).
\end{aligned} \tag{3.21}$$

Then, we obtain from Proposition 3.2 and (3.21) that

$$\begin{aligned}
|\eta(t)-\zeta_p(t)| &\le -\frac{2\sigma}{\lambda}e^{-\lambda\tau_{p+1}}\left(e^{-\lambda(t-\tau_{p+1})}-1\right) \\
&\le -\frac{2\sigma}{\lambda}e^{-\lambda\tau_{p+1}}\left(e^{\frac{-4\lambda}{\sigma-1-\alpha}a_{p+1}}-1\right).
\end{aligned}$$

We now use the inequality $e^x-1\le 2x$ for $0\le x\le 1$, where $x=\frac{-4\lambda}{\sigma-1-\alpha}a_{p+1}$. By (3.1), $x\le\frac{-4\lambda}{\sigma-1-\alpha}a_1\le 1$. Then, using $\tau_{p+1}\le\bar\tau\le T$ (see (3.19)), we have

$$|\eta(t)-\zeta_p(t)| \le \left(\frac{2\sigma}{-\lambda}e^{-\lambda T}\right)\cdot\left(\frac{-8\lambda}{\sigma-1-\alpha}a_{p+1}\right)=\frac{16\sigma e^{-\lambda T}}{\sigma-1-\alpha}a_{p+1},$$

and so (3.20) is satisfied. This completes the proof of the lemma. □

3.4. Estimation of distance between evader and pursuer

We now estimate the distance between the evader η and the pursuer ξ_p.

Lemma 3.2. *Let the evader use strategy* (3.15) *in Case A or* (3.16) *in Case B. Then,*

$$|\eta(t) - \xi_p(t)| > a_{p+1}, \quad \tau_p \le t \le \bar{\tau}. \tag{3.22}$$

P r o o f. Let $t \in [\tau_p, \tau_p'] \cap [\tau_p, \bar{\tau}]$. We use inequalities (3.13) and (3.20) to obtain

$$\begin{aligned} |\eta(t) - \xi_p(t)| &\ge |\xi_p(t) - \zeta_p(t)| - |\zeta_p(t) - \eta(t)| \\ &> \frac{\alpha}{2\sigma} a_p - \frac{16\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1} = \frac{\alpha}{4\sigma} a_p, \end{aligned} \tag{3.23}$$

where we used (see (3.2))

$$a_{p+1} = \beta a_p = \frac{(\sigma - 1 - \alpha)\alpha}{2^6 \sigma^2 e^{-\lambda T}} a_p.$$

Consequently, (3.23) yields that

$$|\eta(t) - \xi_p(t)| > \frac{\alpha}{4\sigma} a_p \ge \beta a_p = a_{p+1}, \quad t \in [\tau_p, \tau_p'] \cap [\tau_p, \bar{\tau}], \tag{3.24}$$

and (3.22) is proved.

Hence, if $\tau_p' > \bar{\tau}$, then $[\tau_p, \tau_p'] \cap [\tau_p, \bar{\tau}] = [\tau_p, \bar{\tau}]$ and (3.22) is true. If $\tau_p' \le \bar{\tau}$, then (3.24) implies that (3.22) is true on $[\tau_p, \tau_p'] \cap [\tau_p, \bar{\tau}] = [\tau_p, \tau_p']$. What remains is to prove (3.22) for $\tau_p' \le t \le \bar{\tau}$ (see Fig. 8).

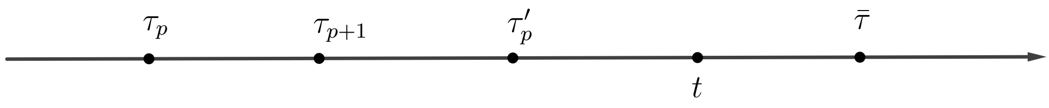

Fig. 8. The case where $\tau_p' \le t \le \bar{\tau}$

Indeed, since $\zeta_{p2}(\tau_p') - \xi_{p2}(\tau_p') > a_p$ by (3.14) and

$$|\eta(\tau_p') - \zeta_p(\tau_p')| \le \frac{16\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1},$$

by (3.20), therefore, in view of $\alpha < \sigma$, we have

$$\begin{aligned} \eta_2(\tau_p') - \xi_{p2}(\tau_p') &= \big(\zeta_{p2}(\tau_p') - \xi_{p2}(\tau_p')\big) + \big(\eta_2(\tau_p') - \zeta_{p2}(\tau_p')\big) \\ &> a_p - |\eta(\tau_p') - \zeta_p(\tau_p')| \ge a_p - \frac{16\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1} \\ &= a_p - \frac{16\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} \cdot \frac{(\sigma - 1 - \alpha)\alpha}{64\sigma^2 e^{-\lambda T}} a_p \\ &= a_p - \frac{\alpha}{4\sigma} a_p > \frac{3}{4} a_p. \end{aligned} \tag{3.25}$$

For $\tau_p' \le t \le \bar{\tau}$, using (3.25), $v_2(s) > 0$, and the fact that $u_{p2}(s) \le 1$, we have

$$\begin{aligned} \eta_2(t) - \xi_{p2}(t) &= \eta_2(\tau_p') - \xi_{p2}(\tau_p') + \int_{\tau_p'}^t e^{-\lambda s} v_2(s)\, ds - \int_{\tau_p'}^t e^{-\lambda s} u_{p2}(s)\, ds \\ &\ge \frac{3}{4} a_p + \int_{\tau_p'}^t e^{-\lambda s} v_2(s)\, ds - \int_{\tau_p'}^t e^{-\lambda s}\, ds \qquad (3.26) \\ &\ge \frac{3}{4} a_p + \frac{1}{\lambda}\big(e^{-\lambda t} - e^{-\lambda \tau_p'}\big). \qquad (3.27) \end{aligned}$$

We now estimate the second summand in (3.32). From (3.17) and Proposition 3.2, we have $t-\tau_p' \le t-\tau_{p+1} \le \frac{4}{\sigma-1-\alpha}a_{p+1}$. Using this and by (3.19), we have

$$-\frac{1}{\lambda}\left(e^{-\lambda t}-e^{-\lambda\tau_p'}\right) = -\frac{1}{\lambda}e^{-\lambda\tau_p'}\left(e^{-\lambda(t-\tau_p')}-1\right) \le -\frac{1}{\lambda}e^{-\lambda T}\left(e^{\frac{-4\lambda}{\sigma-1-\alpha}a_{p+1}}-1\right).$$

Using the relations $e^x-1\le 2x$ for $0\le x\le 1$, where $x=\frac{-4\lambda}{\sigma-1-\alpha}a_{p+1}\le\frac{-4\lambda}{\sigma-1-\alpha}a_1\le 1$ by (3.1), the right-hand side can be easily estimated from above by

$$\left(\frac{e^{-\lambda T}}{-\lambda}\right)\cdot\left(\frac{-8\lambda}{\sigma-1-\alpha}a_{p+1}\right) < \frac{8e^{-\lambda T}}{\sigma-1-\alpha}\cdot\frac{\sigma-1-\alpha}{64e^{-\lambda T}}a_p = \frac{1}{8}a_p,$$

here we used the inequality

$$a_{p+1}=\beta a_p < \frac{\sigma-1-\alpha}{2^6e^{-\lambda T}}a_p$$

following from (3.3).

According to this estimate and the fact that $|\eta(t)-\xi_p(t)|\ge\eta_2(t)-\xi_{p2}(t)$, it follows from (3.27) that

$$|\eta(t)-\xi_p(t)|\ge\eta_2(t)-\xi_{p2}(t) > \frac{3}{4}a_p-\frac{1}{8}a_p > \frac{1}{2}a_p > a_{p+1},\quad \tau_p'\le t\le\bar{\tau}. \tag{3.28}$$

Thus, we conclude that, if $\tau_p'\ge\bar{\tau}$ for the pursuer ξ_p, then, by (3.24),

$$|\eta(t)-\xi_p(t)| > a_{p+1},\quad \tau_p\le t\le\bar{\tau}, \tag{3.29}$$

and if $\tau_p'\le\bar{\tau}$, then combining the inequalities (3.24) and (3.28), we obtain (3.29). Hence, (3.29) is true for each pursuer ξ_p in the group attack in both Cases A and B. The proof of Lemma 3.2 is complete. □

An important conclusion to draw from the inequality (3.29) is that, for $k\ge p+1$, there is no an a_k-approach time τ_k of the pursuer ξ_p to the evader on the time interval $\tau_p<t\le\bar{\tau}$. Indeed, if there was an a_k-approach time τ_k with $\tau_p<\tau_k\le\bar{\tau}$ for some $k\ge p+1$, then we would have had $|\eta(\tau_k)-\xi_p(\tau_k)|=a_k$. However, this is impossible since $a_k\le a_{p+1}$ for $k\ge p+1$ and by (3.29) $|\eta(\tau_k)-\xi_p(\tau_k)|>a_{p+1}$, a contradiction.

Consequently, each pursuer ξ_p has only one approach time τ_p in this group attack on the time interval $\tau_p\le t\le\bar{\tau}$. Therefore, all the pursuers participated in a_k-approaches in the group attack in $[\tau_1,\bar{\tau}]$ are distinct. Thus, Case B is excluded and there are only finite number of pursuers ξ_1, $\xi_2,\ldots,\xi_{k_1}$ with $k_1\le m$ in the first group attack. Hence, we deal with only Case A where $\bar{\tau}=\tau_{k_1}'$ and the first group attack ends at τ_{k_1}'.

The following lemma shows that some pursuers who participated in the first group attack no longer have another an a_k-approach time after τ_{k_1}'.

Lemma 3.3. *Let $\tau_p'\le\tau_{k_1}'$ for the pursuer ξ_p. Then,*

$$\eta_2(t)-\xi_{p2}(t) > a_{p+1},\quad t\ge\tau_{k_1}'. \tag{3.30}$$

P r o o f. Clearly, the inequality $\tau_p'\le\tau_{k_1}'$ is satisfied at least for $p=k_1$, that is, for the pursuer ξ_{k_1}. We now estimate the right-hand side of (3.26), for $t\ge\tau_{k_1}'$. Note that if several group attacks occur on the time interval $[\tau_{k_1}',t)$, then, by the description of evader's strategy (i)–(iii), the evader moves with $v(t)=V_0$ between the group attacks and after the last group attack as well.

Fig. 9 illustrates three group attacks on the intervals $[\tau_1,\tau_{k_1}')$, $[\tau_{k_1+1},\tau_{k_2}')$, $[\tau_{k_2+1},\tau_{k_3}')$.

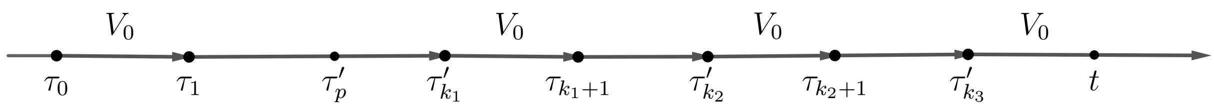

Fig. 9. Evader is under a group attack on the intervals $[\tau_1, \tau'_{k_1})$, $[\tau_{k_1+1}, \tau'_{k_2})$, $[\tau_{k_2+1}, \tau'_{k_3})$

To estimate the integral $\int_{\tau'_p}^t e^{-\lambda s} v_2(s)\,ds$ in (3.26), we use the representation $[\tau'_p, t) = P \cup Q$, $P \cap Q = \emptyset$, where the evader undergoes a group attack of some pursuers on P, and the evader moves with $v(t) = V_0$ on Q. For example, for the interval $[\tau'_p, t)$ in Fig. 9, we have

$$P = [\tau'_p, \tau'_{k_1}) \cup [\tau_{k_1+1}, \tau'_{k_2}) \cup [\tau_{k_2+1}, \tau'_{k_3}), \quad Q = [\tau'_{k_1}, \tau_{k_1+1}) \cup [\tau'_{k_2}, \tau_{k_2+1}) \cup [\tau'_{k_3}, t).$$

Since, by definition of P, each time $t' \in P$, $t' \geq \tau'_p$, (recall $\tau_{p+1} < \tau'_p$) belongs to an interval $[\tau_j, \tau'_j)$ for some $j \geq p+1$, therefore,

$$P \subset \bigcup_{\substack{k \geq p+1, \\ \tau_k \leq t}} [\tau_k, \tau'_k). \tag{3.31}$$

Since $v_2(t) > 0$ for all $t \geq 0$, therefore, $\int_P e^{-\lambda s} v_2(s)\,ds \geq 0$. Using this, we obtain

$$\begin{aligned} \int_{\tau'_p}^t e^{-\lambda s} v_2(s)\,ds &= \int_{P \cup Q} e^{-\lambda s} v_2(s)\,ds = \int_P e^{-\lambda s} v_2(s)\,ds + \int_Q \sigma e^{-\lambda s}\,ds \\ &\geq \sigma \int_Q e^{-\lambda s}\,ds = \sigma \int_{[\tau'_p, t] \setminus P} e^{-\lambda s}\,ds = \sigma \left(\int_{\tau'_p}^t e^{-\lambda s}\,ds - \int_P e^{-\lambda s}\,ds \right). \end{aligned} \tag{3.32}$$

We now estimate the last integral in (3.32). By using (3.31), the inequality (3.8), and $\tau_k \leq \bar{\tau} \leq T$ (see (3.19)), we have

$$\begin{aligned} \int_P e^{-\lambda s}\,ds &\leq \int_{\bigcup\limits_{k \geq p+1} [\tau_k, \tau'_k)} e^{-\lambda s}\,ds \leq \sum_{k \geq p+1} \int_{\tau_k}^{\tau'_k} e^{-\lambda s}\,ds = \sum_{k \geq p+1} \frac{1}{-\lambda} \left(e^{-\lambda \tau'_k} - e^{-\lambda \tau_k} \right) \\ &= \sum_{k \geq p+1} \frac{e^{-\lambda \tau_k}}{-\lambda} \left(e^{-\lambda(\tau'_k - \tau_k)} - 1 \right) \leq \sum_{k \geq p+1} \frac{e^{-\lambda T}}{-\lambda} \left(e^{\frac{-2\lambda a_k}{\sigma - 1 - \alpha}} - 1 \right). \end{aligned} \tag{3.33}$$

Then, using the inequality $e^x - 1 \leq 2x$ for $0 \leq x \leq 1$, and Proposition 3.1, (3.33) can be estimated from above as follows:

$$\int_P e^{-\lambda s}\,ds \leq \sum_{k \geq p+1} \frac{e^{-\lambda T}}{-\lambda} \cdot \frac{-4\lambda a_k}{\sigma - 1 - \alpha} = \frac{8 e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1}.$$

Then, from (3.32), it follows that

$$\int_{\tau'_p}^t e^{-\lambda s} v_2(s)\,ds \geq \sigma \int_{\tau'_p}^t e^{-\lambda s} ds - \frac{8\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1}.$$

Hence, using (3.3), we have from (3.26) that

$$\begin{aligned} \eta_2(t) - \xi_{p2}(t) &\geq \frac{3}{4} a_p + \sigma \int_{\tau'_p}^t e^{-\lambda s}\,ds - \frac{8\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1} - \int_{\tau'_p}^t e^{-\lambda s}\,ds \\ &= \frac{3}{4} a_p - \frac{8\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1} + (\sigma - 1) \int_{\tau'_p}^t e^{-\lambda s}\,ds \\ &\geq \frac{3}{4} a_p - \frac{8\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} a_{p+1} \end{aligned}$$

$$
\begin{aligned}
&> \frac{3}{4}a_p - \frac{8\sigma e^{-\lambda T}}{\sigma - 1 - \alpha} \cdot \frac{\sigma - 1 - \alpha}{32\sigma e^{-\lambda T}} a_p \\
&= \frac{3}{4}a_p - \frac{1}{4}a_p = \frac{1}{2}a_p > a_{p+1},
\end{aligned}
$$

which is the desired conclusion. The proof of the lemma is complete. □

We now prove Theorem 3.1.

P r o o f. Let $\tau'_p \leq \tau'_{k_1}$, $p \leq k_1$ for a pursuer ξ_p in the first group attack of pursuers. Note that this condition is satisfied at least for $p = k_1$. Combining the inequality in (3.4) with $k = p$, inequality (3.22), and inequality (3.30) we obtain that $|\eta(t) - \xi_p(t)| > a_{p+1}$ for all $t \geq 0$. Therefore, the pursuer ξ_p, for which $\tau'_p \leq \tau'_{k_1}$, can never reach the a_{p+1}-vicinity of the evader η. Hence, ξ_p will not participate in the further group attacks starting from the second one satisfying the inequality $\eta_2(t) - \xi_{p2}(t) > a_{p+1} \geq a_{k_1+1}$, $t \geq \tau'_p$.

If the time τ_{k_1+1} occurs, then the evader undergoes the second group attack of some pursuers on an interval $[\tau_{k_1+1}, \tau'_{k_2})$ for some $k_2 \geq k_1 + 1$. We can use similar arguments to obtain $|\eta(t) - \xi_q(t)| > a_{q+1}$ for all $t \geq 0$ and for some $q \in \{k_1 + 1, \ldots, k_2\}$, for which $\tau'_q \leq \tau'_{k_2}$. The pursuer ξ_q will not participate in further group attacks starting from the third one staying "behind" the evader satisfying the inequality $\eta_2(t) - \xi_{q2}(t) > a_{q+1} \geq a_{k_2+1}$, $t \geq \tau'_q$, and so on.

Thus, after the first group attack of pursuers $\xi_1, \xi_2, \ldots, \xi_{k_1}$ we can ignore at least one pursuer, for example, ξ_{k_1}, after the second group attack of pursuers $\xi_{k_1+1}, \xi_{k_1+2}, \ldots, \xi_{k_2}$ we can ignore at least one pursuer from this group of pursuers, for example, ξ_{k_2}, and so on. Since the total number of pursuers is m, therefore, after at most m group attacks of pursuers all the pursuers remain "behind" the evader. The proof of Theorem 3.1 is complete. □

§ 4. Conclusions

We have studied a linear evasion differential game of m pursuers and one evader. The fact that the control sets of the pursuers are the unit ball and that of the evader is the ball of radius σ, $\sigma > 1$, indicate that the evader has an advantage in dynamic capability.

We have proposed an evasion strategy for the evader. Unlike the strategy constructed by [37], the strategy we have developed depends on the parameters of the linear differential equations and approach times τ_i, for which we gave new formulas.

It is important to note that, for the strategy we have constructed, any pursuer who participated in a group attack cannot have another approach time in the same group attack. Therefore, in the first group attack, there are at most m approach times while in the second one there are at most $m-1$ approach times and so on. This fact leads us to one of the main results that the total number of approach times τ_k of m pursuers during the game cannot exceed

$$
m + (m-1) + \cdots + 1 = \frac{m(m+1)}{2}.
$$

It is also worth noting that in the proof of Theorem 3.1, the inequality $\tau'_p \leq \tau'_{k_1}$ plays the key role. If $\tau'_p > \tau'_{k_1}$ for some pursuer ξ_p, then this pursuer can participate in the next group attack.

Next, using the finiteness of the approach times we can estimate the minimum distance between pursuers $\xi_i(t)$ and evader $\eta(t)$, $t \geq 0$. If we let $\tau_{k_{j_0}}$ denote the last approach time, then we have $k_{j_0} \leq m(m+1)/2$ and, therefore,

$$
|\xi_i(t) - \eta(t)| > r = a_{k_{j_0}+1}, \quad t \geq 0, \quad i = 1, 2, \ldots, m.
$$

This means that the evader can avoid capture from all pursuers moving at a distance no less than r from its position. Clearly, r depends on the initial states of the players and parameters α and β.

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Дифференциальная игра уклонения быстрого убегающего от нескольких преследователей

Ключевые слова: линейная дифференциальная игра уклонения, управление, стратегия уклонения, маневр, убегающий, несколько преследователей.

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Мы изучаем линейную дифференциальную игру убегания одного убегающего и m преследователей, $m \geq 2$, в $\mathbb{R}^n$. Множества управлений преследователей — единичные шары, а множество управления убегающего — шар радиуса σ, где $\sigma > 1$. Мы считаем, что уклонение возможно, если состояние убегающего не совпадает с состоянием ни одного из преследователей для всех $t \geq 0$. Для решения задачи уклонения предлагается стратегия для убегающего, и показано, что уклонение возможно из любых заданных начальных положений игроков. Мы покажем, что когда убегающий применяет эту стратегию, максимальное число моментов сближения преследователей с убегающим ограничено сверху выражением $m(m+1)/2$.

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