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CONDITION NUMBERS RELATED TO THE CORE-EP INVERSE

The explicit formulae of the condition numbers for the core-EP inverse and the core-EP inverse solution of a linear system are developed based on the Frobenius norm and the spectral norm, under one-sided assumptions. The structured perturbation for the core-EP inverse is established. Some known results in the literature are particular cases of our results.

Keywords: condition number, core-EP inverse, perturbation, linear system.

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Introduction

As usual, the set of all $m \times n$ complex matrices is denoted by $\mathbb{C}^{m\times n}$ and, for $A \in \mathbb{C}^{m\times n}$, the symbols $R(A)$, $N(A)$, A^* and $\operatorname{rank}(A)$, respectively, represent the range (column space), null space, conjugate transpose and rank of A. If $A \in \mathbb{C}^{n\times n}$, the smallest nonnegative integer k satisfying $\operatorname{rank}(A^{k+1}) = \operatorname{rank}(A^k)$, is the index of A, marked as $\operatorname{ind}(A)$. Also, I_n is the identity matrix of order n.

For $A \in \mathbb{C}^{n\times n}$ with $\operatorname{ind}(A) = k$, there exists the core-EP inverse $A^{\text{\textcircled{†}}} = X \in \mathbb{C}^{n\times n}$ [21] as a unique solution of the system

$$XAX = X \quad \text{and} \quad R(X) = R(X^*) = R(A^k).$$

Especially, if $\operatorname{ind}(A) \le 1$, $A^{\text{\textcircled{†}}} = A^{\text{\textcircled{\#}}}$ is the core inverse of A [2]. The significant of the core-EP inverse was confirmed by the following published papers [1,3,4,7,8,11,12,14,18,19,23,25,29].

The following auxiliary result is useful.

Lemma 0.1 (see [24]). *Let $A \in \mathbb{C}^{n\times n}$, $\operatorname{ind}(A) = k$ and $\operatorname{rank}(A^k) = t$. Then, A can be written as*

$$A = U \begin{bmatrix} A_1 & A_2 \\ 0 & A_3 \end{bmatrix} U^*, \tag{0.1}$$

where U is unitary matrix, A_1 is nonsingular upper-triangular of order t and A_3 is nilpotent of index k. In addition, we have

$$A^{\text{\textcircled{†}}} = U \begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*, \tag{0.2}$$

$$AA^{\text{\textcircled{†}}} = U \begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix} U^*, \quad \textit{and} \quad A^{\text{\textcircled{†}}}A = U \begin{bmatrix} I_t & A_1^{-1}A_2 \\ 0 & 0 \end{bmatrix} U^*.$$

A condition number is a basic notion in numerical analysis for assessing the numerical stability of a problem and predicting the accuracy of its solution [9,10]. The condition number is important in providing a measure of how sensitive the solution of a linear system $Ax = b$ is to small changes in A and b. A high condition number indicates that the solution is highly sensitive to these changes, while a low condition number indicates that the solution is relatively insensitive. For a square and invertible matrix A, the condition number of A is given by $\|A\| \cdot \|A^{-1}\|$, where $\|\cdot\|$ is some matrix norm. The condition number of A can not be introduced in the previous sense when A is rectangular or even square and singular. Then we define the generalized condition number of A related to some generalized inverse A^- of A by $\|A\| \cdot \|A^-\|$.

Generalized condition numbers have applications in studying singular linear systems based on various norms [5,26–28]. Using two-sided conditions, the condition number for the outer inverse was studied in [20] for rectangular matrices, applying the Schur decomposition and the spectral norm and extending results given in [6]. Also, results from [6] are generalized to bounded linear operators in [16].

Utilizing one-sided conditions, the explicit formulae for the condition numbers of the core inverse and the core inverse solution of a linear system were proposed in [15], by means of the Hartwig and Spindelböck's decomposition, the spectral norm and the Frobenius norm.

Since the core inverse exists for square matrices of index 1, the aim of this paper is to extend the results about condition numbers for the core inverse given in [15] to square matrices of arbitrary index. The core-EP inverse presents a generalization of the core inverse for square matrices of arbitrary index and it will be used. More precisely, we establish the strict expression for the condition number with respect to the core-EP inverse, under one-sided conditions. We characterize the Frobenius norm and the spectral norm of the relative condition number for the core-EP inverse, and the level-2 condition number for the core-EP inverse. The sensitivity for the core-EP inverse solution of linear systems is given. The structured perturbation of the core-EP inverse is presented too. Thus, we generalize existing results proposed in [15].

§ 1. Condition numbers related to the core-EP inverse

For $A \in \mathbb{C}^{n\times n}$, $\operatorname{ind}(A) = k$ and $b \in \mathbb{C}^n$, consider the next linear system:

$$Ax = b, \quad x \in R(A^k).$$

The core-EP inverse solution x has the form

$$x = A^{\textcircled{\dagger}} b.$$

The absolute condition number was defined in [22]: when F is a continuously differentiable function

$$F\colon \mathbb{C}^{m\times n} \times \mathbb{C}^m \longrightarrow \mathbb{C}^n$$
$$(A, x) \longmapsto F(A, x),$$

the absolute condition number of F at x is the scalar $\|F'(x)\|$. The relative condition of F at x is

$$\frac{\|F'(x)\|\|x\|}{\|y\|}.$$

We consider the operator

$$F : \mathbb{C}^{n\times n} \times \mathbb{C}^n \longrightarrow \mathbb{C}^n$$
$$(A, b) \longmapsto F(A, b) = A^{\textcircled{\dagger}} b = x.$$

Observe that F is differentiable function, if the perturbation E in A satisfy

$$R(E) \subseteq N((A^k)^*), \tag{1.1}$$

which is equivalent to

$$AA^{\textcircled{\dagger}}E = E. \tag{1.2}$$

According to [17, Corollary 3.1], we have the next perturbation result for the core-EP inverse.

Lemma 1.1 (see [17, Corollary 3.1]). *Let $A \in \mathbb{C}^{n\times n}$ and $\operatorname{ind}(A) = k$. If the perturbation $E \in \mathbb{C}^{n\times n}$ in A satisfies $AA^{\textcircled{\dagger}}E = E$ and $\|A^{\textcircled{\dagger}}E\|_2 < 1$, then*

$$(A+E)^{\textcircled{\dagger}} = (I_n + A^{\textcircled{\dagger}}E)^{-1}A^{\textcircled{\dagger}} = A^{\textcircled{\dagger}}(I_n + EA^{\textcircled{\dagger}})^{-1}.$$

Remark that, by [17, Corollary 3.1], the condition $AA^{\textcircled{\dagger}}E = E$ in Lemma 1.1 can be replaced with $E = A^{\textcircled{\dagger}}AE$ or $E = AA^{\textcircled{\dagger}}EAA^{\textcircled{\dagger}}$ or $E = A^{\textcircled{\dagger}}AEA^{\textcircled{\dagger}}A$.

We take the parameterized weighted Frobenius norm $\|[\lambda A, \mu b]\|^{(F)}_{U,M}$, where U is defined as in (0.1) and $M = \operatorname{diag}(U, 1)$, because we can choose different parameters λ, μ for different perturbations.

Based on the 2-norm and Frobenius norm ($\|B\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|b_{i,j}|^2}$ for $B = [b_{i,j}] \in \mathbb{C}^{m\times n}$), we now obtain the strict expression of the condition number for the core-EP inverse solution. Notice that [15, Theorem 2.2] is a special case of Theorem 1.1 in the case where $\operatorname{ind}(A) \le 1$.

Theorem 1.1. *Let $A \in \mathbb{C}^{n\times n}$ with $\operatorname{ind}(A) = k$ have the form (0.1). If the perturbation E in A satisfies (1.1), the absolute condition number of the core-EP inverse solution of a linear system, with the norm $\|x\|_2$ on the solution and the norm*

$$\|[\lambda A, \mu b]\|^{(F)}_{U,M} = \sqrt{\lambda^2\|A\|_F^2 + \mu^2\|b\|_2^2}$$

on the data (A, b), is

$$C = \|A^{\textcircled{\dagger}}\|_2\sqrt{\frac{1}{\mu^2} + \frac{\|x\|_2^2}{\lambda^2}},$$

where U is the same matrix as in (0.1) and $M = \begin{bmatrix} U & 0 \\ 0 & 1 \end{bmatrix}$.

P r o o f. Clearly, $F(A, b) = A^{\textcircled{\dagger}}b$. Since (1.2) holds, F is a differentiable function and F' is given by

$$F'(A,b)|_{(E,f)} = \lim_{\epsilon\to 0}\frac{(A+\epsilon E)^{\textcircled{\dagger}}(b+\epsilon f) - A^{\textcircled{\dagger}}b}{\epsilon},$$

where f is the perturbation of b and E is the perturbation of A.

By (1.2), it follows

$$(A+\epsilon E)^{\textcircled{\dagger}} = A^{\textcircled{\dagger}} - \epsilon A^{\textcircled{\dagger}}EA^{\textcircled{\dagger}} + O(\epsilon^2),$$

and we show that

$$F'(A,b)|_{(E,f)} = -A^{\textcircled{\dagger}}Ex + A^{\textcircled{\dagger}}f.$$

Further,

$$\begin{aligned}\|F'(A,b)|_{(E,f)}\|_2 &= \|F'(A,b)|_{(E,f)}\|_F = \|A^{\textcircled{\dagger}}(Ex - f)\|_F \\ &\le \|A^{\textcircled{\dagger}}\|_2(\|E\|_F\|x\|_2 + \|f\|_2).\end{aligned}$$

We know that the norm of a linear map $F'(A,b)$ is the supermum of $\|F'(A,b)|_{(E,f)}\|_F$ on the unit ball of $\mathbb{C}^{n\times n}\times\mathbb{C}^n$. From

$$(\|[\lambda E, \mu f]\|^{(F)}_{U,M})^2 = \lambda^2\|E\|_F^2 + \mu^2\|f\|_2^2,$$

we verify that

$$\begin{aligned}
&\|F'(A,b)\| = \\
&= \sup_{\lambda^2\|E\|_F^2+\mu^2\|f\|_2^2=1} \|A^{\textcircled{\dagger}}(Ex-f)\|_F \\
&\leq \sup_{\lambda^2\|E\|_F^2+\mu^2\|f\|_2^2=1} \|A^{\textcircled{\dagger}}\|_2(\|E\|_F\|x\|_2+\|f\|_2) \\
&= \sup_{\lambda^2\|E\|_F^2+\mu^2\|f\|_2^2=1} \|A^{\textcircled{\dagger}}\|_2\left(\lambda\|E\|_F\frac{\|x\|_2}{\lambda}+\mu\|f\|_2\frac{1}{\mu}\right) \\
&= \|A^{\textcircled{\dagger}}\|_2 \sup_{\lambda^2\|E\|_F^2+\mu^2\|f\|_2^2=1} (\lambda\|E\|_F,\mu\|f\|_2)\cdot\left(\frac{\|x\|_2}{\lambda},\frac{1}{\mu}\right),
\end{aligned}$$

where $(\lambda\|E\|_F,\mu\|f\|_2)$ and $\left(\frac{\|x\|_2}{\lambda},\frac{1}{\mu}\right)$ can be seen as vectors in $\mathbb{R}^2$. Using the Cauchy–Schwarz inequality, we conclude that

$$\|F'(A,b)\| \leq \|A^{\textcircled{\dagger}}\|_2\sqrt{\frac{\|x\|_2^2}{\lambda^2}+\frac{1}{\mu^2}}.$$

To prove that this upper bound is attainable, note that there are vectors y and z such that

$$A_1^{-1}y = \|A_1^{-1}\|_2 z = \|A^{\textcircled{\dagger}}\|_2 z, \quad \|y\|_2 = \|z\|_2 = 1.$$

For

$$\hat{y} = U\begin{bmatrix} y \\ 0 \end{bmatrix}, \qquad \hat{z} = U\begin{bmatrix} z \\ 0 \end{bmatrix},$$

we get

$$\|\hat{y}\|_2 = \|\hat{z}\|_2 = 1.$$

Now,

$$\begin{aligned}
A^{\textcircled{\dagger}}\hat{y} &= U\begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix}U^*U\begin{bmatrix} y \\ 0 \end{bmatrix} = U\begin{bmatrix} A_1^{-1}y \\ 0 \end{bmatrix} \\
&= U\begin{bmatrix} \|A_1^{-1}\|_2 z \\ 0 \end{bmatrix} = \|A_1^{-1}\|_2 U\begin{bmatrix} z \\ 0 \end{bmatrix} \\
&= \|A^{\textcircled{\dagger}}\|_2\hat{z}.
\end{aligned}$$

Choosing

$$\eta = \sqrt{\frac{\|x\|_2^2}{\lambda^2}+\frac{1}{\mu^2}}, \qquad f = \frac{1}{\mu^2\eta}\hat{y}, \qquad E = -\frac{1}{\lambda^2\eta}\hat{y}x^*,$$

we obtain

$$\begin{aligned}
AA^{\textcircled{\dagger}}E &= -\frac{1}{\lambda^2\eta}U\begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix}U^*\hat{y}x^* \\
&= -\frac{1}{\lambda^2\eta}U\begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix}U^*U\begin{bmatrix} y \\ 0 \end{bmatrix}x^* \\
&= -\frac{1}{\lambda^2\eta}U\begin{bmatrix} y \\ 0 \end{bmatrix}x^* \\
&= -\frac{1}{\lambda^2\eta}\hat{y}x^* \\
&= E
\end{aligned}$$

and thus E satisfies the assumption (1.2).

In order to check that the perturbation (E, f) is feasible, i. e., $\lambda^2\|E\|_F^2 + \mu^2\|f\|_2^2 = 1$, we observe that

$$x = A^{\textcircled{\dagger}}b = U\begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*b,$$

and

$$\begin{aligned}
\lambda^2\|E\|_F^2 + \mu^2\|f\|_2^2 &= \frac{1}{\lambda^2\eta^2}\|\hat{y}x^*\|_F^2 + \frac{1}{\mu^2\eta^2}\|\hat{y}\|_2^2 \\
&= \frac{1}{\lambda^2\eta^2}\|\hat{y}\|_2^2\|x^*\|_2^2 + \frac{1}{\mu^2\eta^2} \\
&= \frac{1}{\eta^2}\left(\frac{\|x\|_2^2}{\lambda^2} + \frac{1}{\mu^2}\right) \\
&= 1.
\end{aligned}$$

Therefore,

$$\begin{aligned}
F'(A,b)|_{(E,f)} &= -A^{\textcircled{\dagger}}Ex + A^{\textcircled{\dagger}}f \\
&= \frac{1}{\lambda^2\eta}A^{\textcircled{\dagger}}\hat{y}x^*x + \frac{1}{\mu^2\eta}A^{\textcircled{\dagger}}\hat{y} \\
&= \frac{1}{\lambda^2\eta}\|A^{\textcircled{\dagger}}\|_2\hat{z}\|x\|_2^2 + \frac{1}{\mu^2\eta}\|A^{\textcircled{\dagger}}\|_2\hat{z} \\
&= \|A^{\textcircled{\dagger}}\|_2\eta\hat{z}
\end{aligned}$$

and

$$\|F'(A,b)|_{(E,f)}\|_2 = \|A^{\textcircled{\dagger}}\|_2\sqrt{\frac{\|x\|_2^2}{\lambda^2} + \frac{1}{\mu^2}}.$$

with $\lambda^2\|E\|_F^2 + \mu^2\|f\|_2^2 = 1$, which gives

$$\|F'(A,b)\| \geq \|A^{\textcircled{\dagger}}\|_2\sqrt{\frac{\|x\|_2^2}{\lambda^2} + \frac{1}{\mu^2}}.$$

Hence, the proof is finished. □

In order to illustrate Theorem 1.1, we present the next example.

Example 1.1. For

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad E = \begin{bmatrix} \frac{1}{50} & \frac{1}{50} & \frac{1}{50} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

notice that $\operatorname{ind}(A) = 2$ and

$$A^{\textcircled{\dagger}} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Since $\|A^{\textcircled{\dagger}}\|_2 = \frac{1}{2}$, the absolute condition number of the core-EP inverse solution $x = A^{\textcircled{\dagger}}b$ from Theorem 1.1 is

$$C = \frac{1}{2}\sqrt{\frac{1}{\mu^2} + \frac{\|x\|_2^2}{\lambda^2}}.$$

When the assumption (1.1) is satisfied, the 2-norm relative condition number for the core-EP inverse $A^{\oplus}$ is presented by

$$\operatorname{Cond}(A) = \lim_{\epsilon \to 0^+} \sup_{\|E\|_2 \le \epsilon \|A\|_2} \frac{\|(A+E)^{\oplus} - A^{\oplus}\|_2}{\epsilon \|A^{\oplus}\|_2}$$

and the corresponding condition number for the linear systems $Ax = b$ is

$$\operatorname{Cond}(A, b) = \lim_{\epsilon \to 0^+} \sup_{\substack{\|E\|_2 \le \epsilon \|A\|_2 \\ \|f\|_2 \le \epsilon \|b\|_2}} \frac{\|(A+E)^{\oplus}(b+f) - A^{\oplus} b\|_2}{\epsilon \|A^{\oplus} b\|_2}.$$

The level-2 condition number for the core-EP inverse is introduced by

$$\operatorname{Cond}^{[2]}(A) = \lim_{\epsilon \to 0} \sup_{\|E\|_2 \le \epsilon \|A\|_2} \frac{|\operatorname{Cond}(A+E) - \operatorname{Cond}(A)|}{\epsilon \operatorname{Cond}(A)}$$

and the level-2 corresponding condition number is

$$\operatorname{Cond}^{[2]}(A, b) = \lim_{\epsilon \to 0} \sup_{\substack{\|E\|_2 \le \epsilon \|A\|_2 \\ \|f\|_2 \le \epsilon \|b\|_2}} \frac{|\operatorname{Cond}(A+E, b+f) - \operatorname{Cond}(A, b)|}{\epsilon \operatorname{Cond}(A, b)}.$$

We develop an explicit representation for the 2-norm relative condition number of the core-EP inverse.

Theorem 1.2. *Let $A \in \mathbb{C}^{n \times n}$ with $\operatorname{ind}(A) = k$ have the form (0.1). If the perturbation E in A satisfies (1.1), the condition number*

$$\operatorname{Cond}(A) = \lim_{\epsilon \to 0^+} \sup_{\|E\|_2 \le \epsilon \|A\|_2} \frac{\|(A+E)^{\oplus} - A^{\oplus}\|_2}{\epsilon \|A^{\oplus}\|_2}$$

fulfills

$$\operatorname{Cond}(A) = \|A\|_2 \|A^{\oplus}\|_2.$$

P r o o f. By neglecting $\mathcal{O}(\epsilon^2)$ terms in a standard expansion, using Lemma 1.1, we have

$$(A+E)^{\oplus} - A^{\oplus} = -A^{\oplus} E A^{\oplus}.$$

Since $\|E\|_2 \le \epsilon \|A\|_2$, notice that

$$\|(A+E)^{\oplus} - A^{\oplus}\|_2 = \|A^{\oplus} E A^{\oplus}\|_2 \le \epsilon \|A\|_2 \|A^{\oplus}\|_2^2$$

and

$$\frac{\|(A+E)^{\oplus} - A^{\oplus}\|_2}{\epsilon \|A^{\oplus}\|_2} \le \|A\|_2 \|A^{\oplus}\|_2.$$

There exist vectors u and v such that $\|u\|_2 = \|v\|_2 = 1$,

$$\|A_1^{-1} u\|_2 = \|v^* A_1^{-1}\|_2 = \|A_1^{-1}\|_2.$$

For

$$E = \epsilon \|A\|_2 U \begin{bmatrix} u \\ 0 \end{bmatrix} \begin{bmatrix} v^* & 0 \end{bmatrix} U^*,$$

we check that $AA^{\oplus}E = E$ and so E satisfies (1.1). Then,

$$\begin{aligned} \|E\|_2 &= \epsilon\|A\|_2 \left\| U \begin{bmatrix} uv^* & 0 \\ 0 & 0 \end{bmatrix} U^* \right\|_2 = \epsilon\|A\|_2\|uv^*\|_2 \\ &= \epsilon\|A\|_2\|u\|_2\|v\|_2 = \epsilon\|A\|_2, \end{aligned}$$

and

$$\begin{aligned} \|A^{\oplus}EA^{\oplus}\|_2 &= \epsilon\|A\|_2 \left\| U \begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*U \begin{bmatrix} uv^* & 0 \\ 0 & 0 \end{bmatrix} U^*U \begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^* \right\|_2 \\ &= \epsilon\|A\|_2 \left\| U \begin{bmatrix} (A_1^{-1}u)(v^*A_1^{-1}) & 0 \\ 0 & 0 \end{bmatrix} U^* \right\|_2 \\ &= \epsilon\|A\|_2\|A_1^{-1}u\|_2\|v^*A_1^{-1}\|_2 \\ &= \epsilon\|A\|_2\|A_1^{-1}\|_2^2 \\ &= \epsilon\|A\|_2\|A^{\oplus}\|_2^2. \end{aligned}$$

Thus,

$$\frac{\|(A+E)^{\oplus} - A^{\oplus}\|_2}{\epsilon\|A^{\oplus}\|_2} = \|A\|_2\|A^{\oplus}\|_2. \qquad \square$$

The condition number in the Frobenius norm can be studied analogously as Theorem 1.2.

Theorem 1.3. *Let $A \in \mathbb{C}^{n\times n}$ with $\operatorname{ind}(A) = k$ have the form (0.1). If the perturbation E in A satisfies (1.1), the condition number*

$$\operatorname{Cond}_F(A) = \lim_{\epsilon\to 0^+} \sup_{\|E\|_F \le \epsilon\|A\|_F} \frac{\|(A+E)^{\oplus} - A^{\oplus}\|_F}{\epsilon\|A^{\oplus}\|_F}$$

satisfies

$$\operatorname{Cond}_F(A) = \frac{\|A\|_F\|A^{\oplus}\|_2^2}{\|A^{\oplus}\|_F}.$$

P r o o f. Using Lemma 1.1 and by neglecting $\mathcal{O}(\epsilon^2)$ terms in a standard expansion, it follows

$$(A+E)^{\oplus} - A^{\oplus} = -A^{\oplus}EA^{\oplus}.$$

Because $\|E\|_F \le \epsilon\|A\|_F$, we observe that

$$\|(A+E)^{\oplus} - A^{\oplus}\|_F = \|A^{\oplus}EA^{\oplus}\|_F \le \|A^{\oplus}\|_2\|E\|_F\|A^{\oplus}\|_2 \le \epsilon\|A\|_F\|A^{\oplus}\|_2^2$$

and

$$\frac{\|(A+E)^{\oplus} - A^{\oplus}\|_F}{\epsilon\|A^{\oplus}\|_F} \le \frac{\|A\|_F\|A^{\oplus}\|_2^2}{\|A^{\oplus}\|_F}.$$

Let

$$E = \epsilon\|A\|_F U \begin{bmatrix} u \\ 0 \end{bmatrix} \begin{bmatrix} v^* & 0 \end{bmatrix} U^*,$$

where $\|A_1^{-1}u\|_2 = \|v^*A_1^{-1}\|_2 = \|A_1^{-1}\|_2$ and $\|u\|_2 = \|v\|_2 = 1$. Now, we get $AA^{\oplus}E = E$, i.e., E satisfies (1.1),

$$\begin{aligned} \|E\|_F &= \epsilon\|A\|_F \left\| U \begin{bmatrix} uv^* & 0 \\ 0 & 0 \end{bmatrix} U^* \right\|_F \\ &= \epsilon\|A\|_F\|u\|_2\|v\|_2 \\ &= \epsilon\|A\|_F, \end{aligned}$$

and

$$\begin{aligned}\|A^{\textcircled{\dagger}}EA^{\textcircled{\dagger}}\|_F &= \epsilon\|A\|_F\left\|U\begin{bmatrix} A_1^{-1} & 0\\ 0 & 0\end{bmatrix}U^*U\begin{bmatrix} uv^* & 0\\ 0 & 0\end{bmatrix}U^*U\begin{bmatrix} A_1^{-1} & 0\\ 0 & 0\end{bmatrix}U^*\right\|_F\\ &= \epsilon\|A\|_F\left\|U\begin{bmatrix} (A_1^{-1}u)(v^*A_1^{-1}) & 0\\ 0 & 0\end{bmatrix}U^*\right\|_F\\ &= \epsilon\|A\|_F\|A_1^{-1}u\|_2\|v^*A_1^{-1}\|_2\\ &= \epsilon\|A\|_F\|A_1^{-1}\|_2^2\\ &= \epsilon\|A\|_F\|A^{\textcircled{\dagger}}\|_2^2.\end{aligned}$$

We complete the proof by

$$\frac{\|(A+E)^{\textcircled{\dagger}}-A^{\textcircled{\dagger}}\|_F}{\epsilon\|A^{\textcircled{\dagger}}\|_F}=\frac{\|A\|_F\|A^{\textcircled{\dagger}}\|_2^2}{\|A^{\textcircled{\dagger}}\|_F}.$$ □

The following example illustrates Theorem 1.2 and Theorem 1.3.

Example 1.2. Let A and E be given as in Example 1.1. Using $\|A\|_2=2.236$, $\|A\|_F=2.449$ and $\|A^{\textcircled{\dagger}}\|_F=\|A^{\textcircled{\dagger}}\|_2=\frac{1}{2}$, the condition numbers defined in Theorem 1.2 and Theorem 1.3 are

$$\mathrm{Cond}(A)=\|A\|_2\|A^{\textcircled{\dagger}}\|_2=1.118$$

and

$$\mathrm{Cond}_F(A)=\frac{\|A\|_F\|A^{\textcircled{\dagger}}\|_2^2}{\|A^{\textcircled{\dagger}}\|_F}=1.2245.$$

The condition number of linear systems is characterized now in terms with 2-norm.

Theorem 1.4. *Let $A\in\mathbb{C}^{n\times n}$ with $\mathrm{ind}(A)=k$ have the form (0.1). If the perturbation E in A satisfies (1.1), the condition number of linear systems $Ax=b$, $x\in R(A^k)$,*

$$\mathrm{Cond}(A,b)=\lim_{\epsilon\to 0^+}\sup_{\substack{\|E\|_2\le\epsilon\|A\|_2\\ \|f\|_2\le\epsilon\|b\|_2}}\frac{\|(A+E)^{\textcircled{\dagger}}(b+f)-A^{\textcircled{\dagger}}b\|_2}{\epsilon\|A^{\textcircled{\dagger}}b\|_2}$$

fulfills

$$\mathrm{Cond}(A,b)=\|A\|_2\|A^{\textcircled{\dagger}}\|_2+\frac{\|A^{\textcircled{\dagger}}\|_2\|b\|_2}{\|A^{\textcircled{\dagger}}b\|_2}.$$

P r o o f. If $\|E\|_2\le\epsilon\|A\|_2$ and $\|f\|_2\le\epsilon\|b\|_2$, then, by Lemma 1.1,

$$\begin{aligned}(A+E)^{\textcircled{\dagger}}(b+f)-A^{\textcircled{\dagger}}b &= [(A+E)^{\textcircled{\dagger}}-A^{\textcircled{\dagger}}]b+(A+E)^{\textcircled{\dagger}}f\\ &= -A^{\textcircled{\dagger}}EA^{\textcircled{\dagger}}b+(A+E)^{\textcircled{\dagger}}f\\ &= -A^{\textcircled{\dagger}}Ex+A^{\textcircled{\dagger}}f+\mathcal{O}(\epsilon^2)\end{aligned}$$

and so

$$\begin{aligned}\|(A+E)^{\textcircled{\dagger}}(b+f)-A^{\textcircled{\dagger}}b\|_2 &\le \|A^{\textcircled{\dagger}}\|_2\|E\|_2\|x\|_2+\|A^{\textcircled{\dagger}}\|_2\|f\|_2\\ &\le \epsilon\|A^{\textcircled{\dagger}}\|_2(\|A\|_2\|x\|_2+\|b\|_2).\end{aligned}$$

Therefore,

$$\mathrm{Cond}(A,b)\le\|A\|_2\|A^{\textcircled{\dagger}}\|_2+\frac{\|A^{\textcircled{\dagger}}\|_2\|b\|_2}{\|A^{\textcircled{\dagger}}b\|_2}.$$

Set $u = U\begin{bmatrix} v \\ 0 \end{bmatrix}$, for $\|A_1^{-1}v\|_2 = \|A_1^{-1}\|_2$ and $\|v\|_2 = 1$. Further, $\|u\|_2 = 1$ and

$$\begin{aligned} \|A^{\textcircled{\dagger}}u\|_2 &= \left\| U\begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*U\begin{bmatrix} v \\ 0 \end{bmatrix} \right\|_2 \\ &= \|A_1^{-1}v\|_2 \\ &= \|A^{\textcircled{\dagger}}\|_2. \end{aligned}$$

Let $U^* = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$, where $U_1 \in \mathbb{C}^{t\times n}$. Since

$$U^*x = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} x = \begin{bmatrix} U_1x \\ U_2x \end{bmatrix}$$

and

$$U^*x = U^*A^{\textcircled{\dagger}}b = U^*U\begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*b = \begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*b \equiv \begin{bmatrix} s \\ 0 \end{bmatrix},$$

$U_1x = s$ and $U_2x = 0$. From $x \neq 0$, $U_1x \neq 0$ and $\|U^*x\|_2 = \|U_1x\|_2$. Taking

$$f = \epsilon u\|b\|_2, \quad E = -\frac{\epsilon\|A\|_2}{\|U^*x\|_2} ux^*U\begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix} U^*,$$

we check that $AA^{\textcircled{\dagger}}E = E$,

$$\|f\|_2 = \epsilon\|b\|_2\|u\|_2 = \epsilon\|b\|_2$$

and

$$\begin{aligned} \|E\|_2 &= \frac{\epsilon\|A\|_2}{\|U^*x\|_2} \left\| ux^*U\begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix} U^* \right\|_2 \\ &= \frac{\epsilon\|A\|_2}{\|U^*x\|_2} \left\| U\begin{bmatrix} v \\ 0 \end{bmatrix} x^*U\begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix} U^* \right\|_2 \\ &= \frac{\epsilon\|A\|_2}{\|U^*x\|_2} \left\| \begin{bmatrix} v \\ 0 \end{bmatrix} \left(x^*U\begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix} \right) \right\|_2 \\ &= \frac{\epsilon\|A\|_2}{\|U^*x\|_2} \|v\|_2 \left\| x^*U\begin{bmatrix} I_t & 0 \\ 0 & 0 \end{bmatrix} \right\|_2 \\ &= \frac{\epsilon\|A\|_2}{\|U^*x\|_2} \left\| \begin{bmatrix} (U_1x)^* & 0 \end{bmatrix} \right\|_2 \\ &= \frac{\epsilon\|A\|_2}{\|U^*x\|_2} \|U_1x\|_2 \\ &= \epsilon\|A\|_2. \end{aligned}$$

Now,

$$\begin{aligned} \|(A+E)^{\textcircled{\dagger}}(b+f) - A^{\textcircled{\dagger}}b\|_2 &= \|-A^{\textcircled{\dagger}}Ex + A^{\textcircled{\dagger}}f\|_2 \\ &= \left\| \frac{\epsilon\|A\|_2\|U_1x\|_2^2}{\|U^*x\|_2} A^{\textcircled{\dagger}}u + \epsilon\|b\|_2A^{\textcircled{\dagger}}u \right\|_2 \\ &= \epsilon(\|A\|_2\|U^*x\|_2 + \|b\|_2)\|A^{\textcircled{\dagger}}u\|_2 \\ &= \epsilon(\|A\|_2\|A^{\textcircled{\dagger}}b\|_2 + \|b\|_2)\|A^{\textcircled{\dagger}}\|_2, \end{aligned}$$

which gives

$$\frac{\|(A+E)^{\oplus}(b+f)-A^{\oplus}b\|_2}{\epsilon\|A^{\oplus}b\|_2}=\|A\|_2\|A^{\oplus}\|_2+\frac{\|A^{\oplus}\|_2\|b\|_2}{\|A^{\oplus}b\|_2}.$$

The proof is completed. □

The following result in Frobenius norm can be shown in the similar manner as Theorem 1.4.

Theorem 1.5. *Let $A\in\mathbb{C}^{n\times n}$ with $\operatorname{ind}(A)=k$ have the form (0.1). If the perturbation E in A satisfies (1.1), the condition number of linear systems $Ax=b$*

$$\operatorname{Cond}_F(A,b)=\lim_{\epsilon\to 0^+}\sup_{\substack{\|E\|_F\le\epsilon\|A\|_F\\ \|f\|_F\le\epsilon\|b\|_F}}\frac{\|(A+E)^{\oplus}(b+f)-A^{\oplus}b\|_F}{\epsilon\|A^{\oplus}b\|_F}$$

fulfills

$$\operatorname{Cond}_F(A,b)=\|A\|_F\|A^{\oplus}\|_2+\frac{\|A^{\oplus}\|_2\|b\|_2}{\|A^{\oplus}b\|_2}.$$

We need the next results in order to prove that the sensitivity of the condition number for the core-EP inverse in solving a linear system, is approximately given by the condition number itself.

Lemma 1.2. *Let $A\in\mathbb{C}^{n\times n}$ with $\operatorname{ind}(A)=k$ have the form (0.1). For $\hat{y},\hat{z}$ given as in Theorem 1.1, there exists $D\in\mathbb{C}^{n\times n}$ satisfying*

$$D\hat{z}=-\hat{y},\quad \|D\|_2=1,\quad AA^{\oplus}D=D.$$

P r o o f. If $D=-\hat{y}\hat{z}^*$, then $D\hat{z}=-\hat{y}\hat{z}^*\hat{z}=-\hat{y}\|\hat{z}\|_2^2=-\hat{y}$,

$$\|D\|_2=\|\hat{y}\hat{z}^*\|_2=\|\hat{y}\|_2\|\hat{z}\|_2=1$$

and

$$AA^{\oplus}D=-U\begin{bmatrix} I_t & 0\\ 0 & 0\end{bmatrix}U^*\hat{y}\hat{z}^*=-U\begin{bmatrix} y\\ 0\end{bmatrix}\hat{z}^*=D.$$ □

Lemma 1.3. *Let $A\in\mathbb{C}^{n\times n}$ with $\operatorname{ind}(A)=k$ have the form (0.1). If $\epsilon\to 0$, then*

$$\max_{\|E\|_2\le\epsilon\|A\|_2}\Big|\|(A+E)^{\oplus}\|_2-\|A^{\oplus}\|_2\Big|=\epsilon\|A^{\oplus}\|_2\operatorname{Cond}(A)+\mathcal{O}(\epsilon^2),$$

when E satisfies the assumption (1.1).

P r o o f. The fact that E satisfies (1.1) implies

$$(A+E)^{\oplus}=A^{\oplus}-A^{\oplus}EA^{\oplus}+\mathcal{O}(\epsilon^2)$$

and

$$\max_{\|E\|_2\le\epsilon\|A\|_2}\Big|\|(A+E)^{\oplus}\|_2-\|A^{\oplus}\|_2\Big|\le\epsilon\|A^{\oplus}\|_2\operatorname{Cond}(A)+\mathcal{O}(\epsilon^2).$$

For D as in Lemma 1.2 and $E=\epsilon\|A\|_2D$, we get

$$\begin{aligned}\|A^{\oplus}-A^{\oplus}EA^{\oplus}\|_2&\ge\|(A^{\oplus}-A^{\oplus}EA^{\oplus})\hat{y}\|_2=\|A^{\oplus}\hat{y}-A^{\oplus}EA^{\oplus}\hat{y}\|_2\\&=\|A^{\oplus}\hat{y}-\epsilon\|A\|_2A^{\oplus}DA^{\oplus}\hat{y}\|_2\\&=\Big\|\|A^{\oplus}\|_2\hat{z}-\epsilon\|A\|_2\|A^{\oplus}\|_2A^{\oplus}D\hat{z}\Big\|_2\\&=\|A^{\oplus}\|_2\Big\|\hat{z}+\epsilon\|A\|_2A^{\oplus}\hat{y}\Big\|_2\\&=\|A^{\oplus}\|_2\Big\|\hat{z}+\epsilon\|A\|_2\|A^{\oplus}\|_2\hat{z}\Big\|_2\\&=\|A^{\oplus}\|_2\Big(1+\epsilon\|A\|_2\|A^{\oplus}\|_2\Big).\end{aligned}$$ □

We easy prove the following results.

Corollary 1.1. *Let A and E be given as in Lemma 1.1. If the perturbation E in A satisfies (1.1), the level-2 condition number*

$$\operatorname{Cond}^{[2]}(A) = \lim_{\epsilon\to 0} \sup_{\|E\|_2\le\epsilon\|A\|_2} \frac{|\operatorname{Cond}(A+E)-\operatorname{Cond}(A)|}{\epsilon\operatorname{Cond}(A)}$$

fulfills

$$|\operatorname{Cond}^{[2]}(A)-\operatorname{Cond}(A)|\le 1.$$

Corollary 1.2. *Let A and E be given as in Lemma 1.1. If the perturbation E in A satisfies (1.1), the level-2 condition number of linear systems $Ax=b$, $x\in R(A^k)$,*

$$\operatorname{Cond}^{[2]}(A,b) = \lim_{\epsilon\to 0} \sup_{\substack{\|E\|_2\le\epsilon\|A\|_2\\ \|f\|_2\le\epsilon\|b\|_2}} \frac{|\operatorname{Cond}(A+E,b+f)-\operatorname{Cond}(A,b)|}{\epsilon\operatorname{Cond}(A,b)}$$

fulfills

$$\frac{\operatorname{Cond}(A,b)}{(1+\delta)^2}-\frac{1}{1+\delta}\le \operatorname{Cond}^{[2]}(A,b)\le 3\operatorname{Cond}(A,b)+2,$$

where $\delta=\frac{\|b\|_2}{\|AA^{\textcircled{\dagger}}b\|_2}$.

Notice that the hypothesis that E in A satisfies (1.1) in the previous results can be replaced with

$$R(E)\subseteq R(A^k)$$

or equivalently

$$A^{\textcircled{\dagger}}AE=E.$$

Also, we can use $E=AA^{\textcircled{\dagger}}EAA^{\textcircled{\dagger}}$ or $E=A^{\textcircled{\dagger}}AEA^{\textcircled{\dagger}}A$ instead of (1.1).

§ 2. Structured perturbation

In order to present a structured perturbation for the core-EP inverse in 2–norm, for $A=(a_{i,j})$ and $B=(b_{i,j})$, denote by $|A|\le|B|$ if $|a_{i,j}|\le|b_{i,j}|$.

Theorem 2.1. *Let $A\in\mathbb{C}^{n\times n}$ and $\operatorname{ind}(A)=k$. If $\|A^{\textcircled{\dagger}}E\|_2<1$ and $|U^*EU|\le|U^*AA^{\textcircled{\dagger}}AU|$, then*

$$(A+E)^{\textcircled{\dagger}}=(I_n+A^{\textcircled{\dagger}}E)^{-1}A^{\textcircled{\dagger}},$$

where U is given by (0.1).

P r o o f. If $E=U\begin{bmatrix}E_1 & E_2\\ E_3 & E_4\end{bmatrix}U^*$, using (0.1), (0.2) and $|U^*EU|\le|U^*AA^{\textcircled{\dagger}}AU|$, it follows that

$$\left|\begin{bmatrix}E_1 & E_2\\ E_3 & E_4\end{bmatrix}\right|\le\left|\begin{bmatrix}A_1 & A_2\\ 0 & 0\end{bmatrix}\right|$$

yielding $E_3=0$ and $E_4=0$. So, $E=U\begin{bmatrix}E_1 & E_2\\ 0 & 0\end{bmatrix}U^*$ and

$$A+E=U\begin{bmatrix}A_1+E_1 & A_2+E_2\\ 0 & A_3\end{bmatrix}U^*.$$

Since $\|A^{\oplus}E\|_2 < 1$,

$$I_n + A^{\oplus}E = U \begin{bmatrix} A_1^{-1}(A_1 + E_1) & A_1^{-1}E_2 \\ 0 & I_{n-t} \end{bmatrix} U^*$$

is invertible. We deduce that $A_1^{-1}(A_1 + E_1)$ is invertible and thus $A_1 + E_1$ is also invertible. Applying [13, Lemma 2.3], we obtain

$$(A + E)^{\oplus} = U \begin{bmatrix} (A_1 + E_1)^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*.$$

Therefore,

$$\begin{aligned} (I_n + A^{\oplus}E)^{-1}A^{\oplus} &= U \begin{bmatrix} (A_1 + E_1)^{-1}A_1 & -(A_1 + E_1)^{-1}E_2 \\ 0 & I_{n-t} \end{bmatrix} \begin{bmatrix} A_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^* \\ &= U \begin{bmatrix} (A_1 + E_1)^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^* \\ &= (A + E)^{\oplus}. \end{aligned}$$

□

§ 3. Conclusion

Recent results about the condition numbers for the core inverse motivated us to further develop this topic using the core-EP inverse, because the core inverse exists for square matrices of index 1 and the core-EP inverse for square matrices of arbitrary index. Under one-sided conditions, we present strict formulae for the condition numbers of the core-EP inverse and the core-EP inverse solution of a linear system. Thus, we extend existing results in [15] to a more general case. For future researches, it is interesting to generalize these results to tensors or to bounded linear operators on Hilbert spaces or to elements of C^*-algebra.

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Числа обусловленности, связанные с ядерной EP обратной матрицей

Ключевые слова: число обусловленности, ядерная EP обратная матрица, возмущение, линейная система.

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На основе нормы Фробениуса и спектральной нормы, при односторонних предположениях, выведены явные формулы для чисел обусловленности ядерной EP обратной матрицы и решения линейной системы, полученного с помощью этой матрицы. Исследовано поведение ядерной EP обратной матрицы при структурных возмущениях. Показано, что некоторые известные результаты, представленные в литературе, являются частными случаями полученных нами результатов.

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