VESTNIK UDMURTSKOGO UNIVERSITETA. MATEMATIKA. MEKHANIKA. KOMP'YUTERNYE NAUKI
MATHEMATICS 2026. Vol. 36. Issue 3. Pp. 486–495.

MSC2020: 53C21, 53C25, 58J05

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RIGIDITY OF COMPACT RIEMANN SOLITONS VIA THE AHLFORS LAPLACIAN

A Riemann soliton is a rigid generalization of the classical Ricci soliton in which the full Riemann curvature tensor enters the defining equation. We establish a connection between compact Riemann solitons and the operator framework of the Ahlfors Laplacian. Contracting the Riemann soliton equation in dimension $m \geqslant 3$ produces an associated Ricci almost soliton with a normalized potential vector field. We derive a global formula expressing the soliton constant in terms of the average scalar curvature of the compact manifold. Using the Cauchy–Ahlfors operator and its formal adjoint, we obtain rigidity conditions under which the traceless Ricci tensor vanishes and the metric is Einstein. An application of Obata's theorem gives a spherical rigidity result. The two-dimensional case is treated separately: positive constant scalar curvature on an orientable compact Riemann soliton yields a round two-sphere with the corresponding constant Gaussian curvature, while the Ricci almost soliton associated by contraction is trivial.

Keywords: Riemann soliton, Ricci almost soliton, Ahlfors Laplacian, Cauchy–Ahlfors operator, rigidity, Einstein manifold.

DOI: 10.35634/vm260309

Introduction

The theory of geometric flows, initiated by R. Hamilton (see [1]) through the introduction of the Ricci flow, has profoundly influenced modern differential geometry and its applications to mathematical physics. Self-similar solutions of such flows, usually called solitons, play a fundamental role as canonical models for the formation of singularities and for the analysis of the geometric and topological structure of manifolds.

In 2016, E. Hirică and C. Udrişte introduced in [2] a more rigid and structurally richer notion, known as a Riemann soliton (M, g, V, λ). Whereas a classical Ricci soliton imposes a condition only on the Ricci tensor, that is, on a contraction of the curvature tensor, a Riemann soliton involves the full $(0,4)$-tensor Riemann curvature equation. This makes the corresponding soliton equation considerably stronger and places it naturally within the framework of curvature evolution governed by the Kulkarni–Nomizu product. Recent investigations have also revealed significant interactions between Riemann solitons, Ricci-type solitons, and contact geometric structures. Examples include contact metric manifolds, (α, β)-contact geometries, and almost Riemann solitons, where various rigidity and classification phenomena have been established [3–5].

In addition to its intrinsic geometric interest, the theory of Riemann solitons has recently attracted attention in mathematical physics and general relativity. In particular, Riemann solitons have been studied on perfect and imperfect fluid spacetimes, including generalized Robertson–Walker settings, and in modified gravity models such as energy–momentum squared $f(r, T^2)$-gravity. In these settings, they provide a useful geometric framework for describing self-similar configurations, curvature properties, energy conditions, and related features of gravitational models (see [6–8]).

A fundamental algebraic feature of a Riemann soliton (g, V, λ) on an m-dimensional manifold is that, for $m \geqslant 3$, contraction of its defining equation gives rise to a Ricci almost soliton structure (g, X, σ), where the normalized potential vector field is $X = \frac{m-2}{2}V$ and the soliton function σ depends on $\operatorname{div} V$. Thus, every Riemann soliton in dimension $m \geqslant 3$ canonically determines a Ricci almost soliton structure (see [9]).

Since the traceless Ricci tensor Ric_0 plays a central role in the study of Ricci almost solitons, it is natural to ask whether the Ahlfors Laplacian framework developed in [9] can be applied to Riemann solitons. The main purpose of the present paper is to show that the Ahlfors Laplacian provides an effective analytic tool for studying compact Riemann solitons. We first show that the soliton constant λ is determined by the average scalar curvature. We then combine the contracted soliton equation with the Ahlfors operator to obtain rigidity results forcing Ric_0 to vanish under natural assumptions. Consequently, several classes of compact Riemann solitons reduce to Einstein metrics.

§ 1. Preliminaries and the Tachibana–Ahlfors operator framework

Let (M, g) be an m-dimensional smooth Riemannian manifold. We denote by R the Riemann curvature tensor of type $(0, 4)$, whose local components are R_{ijkl}.

Recall that the Kulkarni–Nomizu product $\odot$ of two symmetric $(0, 2)$-tensors A and B is the $(0, 4)$-tensor defined by

$$(A \odot B)(X, Y, Z, W) = A(X, Z)B(Y, W) + A(Y, W)B(X, Z) \\ - A(X, W)B(Y, Z) - A(Y, Z)B(X, W).$$

Following Hirică and Udrişte, a triple (g, V, λ) on M is called a Riemann soliton if it satisfies the structural equation (see [2])

$$2R + \lambda(g \odot g) + g \odot \mathcal{L}_V g = 0. \tag{1.1}$$

Here $\mathcal{L}_V g$ denotes the Lie derivative of the metric tensor along a smooth vector field V, and λ is a real constant.

To investigate equation (1.1), we employ an operator-theoretic approach based on the theory of conformal Killing forms and Tachibana operators.

Let S^2M denote the bundle of symmetric $(0, 2)$-tensors on (M, g). The conformal Killing operator S, also called the Cauchy–Ahlfors operator, maps a 1-form $\theta = X^\flat$, the metric dual of a vector field X, to the space of trace-free symmetric tensors via

$$S\theta = \mathcal{L}_X g - \frac{2}{m}(\operatorname{div} X)g. \tag{1.2}$$

This trace-free symmetrized gradient is the conformal Killing, or Cauchy–Ahlfors, operator; see, for example, Branson [10]. A vector field X is conformal if and only if S annihilates its metric-dual one-form $\theta = X^\flat$.

Remark 1.1. In this paper we follow the normalization of the Cauchy–Ahlfors operator used by Branson [10].

The formal adjoint of the Cauchy–Ahlfors operator is $S^\star = 2\delta$ on symmetric traceless tensors. The composition $S^\star S$ defines the Ahlfors Laplacian; see [10–12]:

$$\Delta_A := S^\star S = 2\delta d + \frac{4(m-1)}{m} d\delta - 4\,\mathrm{Ric}\,.$$

Here d is the exterior derivative on differential forms and δ is its formal adjoint.

Remark 1.2. The Ahlfors Laplacian is distinct from the classical rough and Hodge–de Rham Laplacians. It may be viewed as the $p = 1$ case of the elliptic Tachibana operator acting on differential p-forms; see [9, 10, 13]. The analytical properties of the Ahlfors Laplacian have also been studied in various settings; see, for example, Branson [10], Branson et al. [11], and Pierzchalski and Ørsted [12].

Furthermore, according to the fundamental geometric decomposition, the traceless part of the Ricci tensor

$$\operatorname{Ric}_0 = \operatorname{Ric} - \frac{s}{m} g$$

can be uniquely represented on a compact Riemannian manifold as the L^2-orthogonal sum (see [9])

$$\operatorname{Ric}_0 = S\eta + \operatorname{Ric}_{TT},$$

where η is a smooth 1-form and Ric_{TT} is a transverse-traceless tensor satisfying

$$S^\star \operatorname{Ric}_{TT} = 0.$$

This decomposition separates the conformal component of the traceless Ricci tensor from its transverse-traceless part and provides a direct link between curvature and the kernel of the adjoint Ahlfors operator. Combined with the contracted Riemann soliton equation, it translates the original curvature-tensor condition into an elliptic operator equation.

§ 2. Main rigidity and trivialization theorems

We begin with the contraction of the Riemann soliton equation

$$2R + \lambda(g \odot g) + g \odot \mathcal{L}_V g = 0. \tag{2.1}$$

Writing out the Kulkarni–Nomizu product in (2.1) and contracting over one pair of indices gives, for $m \geqslant 3$,

$$\frac{1}{2}\mathcal{L}_V g + \frac{1}{m-2} \operatorname{Ric} = -\frac{(m-1)\lambda + \operatorname{div} V}{m-2} g.$$

Multiplying by $m-2$, we obtain

$$\frac{m-2}{2}\mathcal{L}_V g + \operatorname{Ric} = \big(-(m-1)\lambda - \operatorname{div} V\big) g.$$

Using the linearity $\mathcal{L}_{cV} g = c\mathcal{L}_V g$, introduce the normalized vector field

$$X = \frac{m-2}{2} V,$$

so that $\mathcal{L}_X g = \frac{m-2}{2}\mathcal{L}_V g$ and $\operatorname{div} V = \frac{2}{m-2} \operatorname{div} X$. Substitution into the contracted equation gives the Ricci almost soliton equation

$$\operatorname{Ric} + \mathcal{L}_X g = \sigma g, \tag{2.2}$$

where

$$\sigma = -(m-1)\lambda - \frac{2}{m-2} \operatorname{div} X, \qquad m \geqslant 3.$$

Taking the traceless part of (2.2) and using (1.2) yields

$$\operatorname{Ric}_0 = -S\theta, \qquad \theta = X^\flat.$$

Thus, in the general Ahlfors decomposition $\operatorname{Ric}_0 = S\eta + \operatorname{Ric}_{TT}$, the soliton equation corresponds to $\eta = -\theta$ and $\operatorname{Ric}_{TT} = 0$.

Taking the trace of (2.2), we obtain

$$s + \frac{4(m-1)}{m-2} \operatorname{div} X = -m(m-1)\lambda.$$

Integrating over the compact manifold M and using the divergence theorem gives

$$\int_M \operatorname{div} X \, dv_g = 0,$$

and therefore

$$\int_M s \, dv_g = -m(m-1)\lambda \operatorname{Vol}(M,g).$$

Consequently,

$$\lambda = -\frac{1}{m(m-1)} \frac{\int_M s \, dv_g}{\operatorname{Vol}(M,g)}.$$

Writing $\overline{s}$ for the average scalar curvature,

$$\overline{s} = \frac{1}{\operatorname{Vol}(M,g)} \int_M s \, dv_g,$$

we obtain the following result.

Proposition 2.1. *Let (M,g,V,λ) be an m-dimensional $(m \geqslant 3)$ compact Riemann soliton. Then, the soliton constant is*

$$\lambda = -\frac{\overline{s}}{m(m-1)},$$

where $\overline{s}$ is the average scalar curvature of (M,g).

Under the sign convention adopted for equation (1.1), the following statement is immediate.

Corollary 2.1. *A compact Riemann soliton is shrinking, steady, or expanding according as its average scalar curvature is positive, zero, or negative, respectively.*

We now consider the gradient case. Let $X = \nabla f$, where X is the normalized potential vector field. Then, the contracted soliton equation yields a pointwise relation between the scalar curvature and the Laplacian of f.

Proposition 2.2. *Let (M,g,V,λ) be an m-dimensional $(m \geqslant 3)$ gradient Riemann soliton, and let X denote its normalized potential vector field. Assume that $X = \nabla f$ for some $f \in C^\infty(M)$. Then,*

$$s + \frac{4(m-1)}{m-2}\Delta f = -m(m-1)\lambda.$$

P r o o f. By definition of the normalized potential vector field,

$$X = \frac{m-2}{2}V.$$

The contraction of the Riemann soliton equation takes the form (2.2), where

$$\sigma = -(m-1)\lambda - \frac{2}{m-2}\operatorname{div} X.$$

Since $X = \nabla f$, we have $\mathcal{L}_X g = 2\nabla^2 f$ and $\operatorname{div} X = \Delta f$. Taking the metric trace gives

$$s + 2\Delta f = m\sigma.$$

Substituting the expression for σ, we obtain

$$s + 2\Delta f = -m(m-1)\lambda - \frac{2m}{m-2}\Delta f.$$

Hence

$$s + \left(2 + \frac{2m}{m-2}\right) \Delta f = -m(m-1)\lambda.$$

Since

$$2 + \frac{2m}{m-2} = \frac{4(m-1)}{m-2},$$

the asserted identity follows. □

Proposition 2.3. *Let (M, g, V, λ) be an m-dimensional $(m \geqslant 3)$ compact Riemann soliton, let $X = \frac{m-2}{2}V$, and put $\theta = X^\flat$. Then,*

$$S^\star S\theta = \frac{m-2}{m} ds.$$

Consequently,

$$\langle S\theta, S\theta\rangle_{L^2} = \frac{m-2}{m} \int_M \mathcal{L}_X s \, dv_g.$$

In particular, if s is constant or $\int_M \mathcal{L}_X s \, dv_g = 0$, then $S\theta = 0$. Hence, X is conformal and, by $\mathrm{Ric}_0 = -S\theta$, the metric g is Einstein.

P r o o f. From $\mathrm{Ric}_0 = -S\theta$, application of $S^\star$ gives

$$-S^\star S\theta = S^\star \mathrm{Ric}_0 .$$

Since $S^\star = 2\delta$ on symmetric traceless tensors,

$$S^\star \mathrm{Ric}_0 = 2\delta \mathrm{Ric}_0 .$$

Using $\mathrm{Ric}_0 = \mathrm{Ric} - \frac{s}{m} g$, the contracted second Bianchi identity $\delta \mathrm{Ric} = -\frac{1}{2} ds$, and $\delta(sg) = -ds$, we obtain

$$\delta \mathrm{Ric}_0 = -\frac{1}{2} ds + \frac{1}{m} ds = -\frac{m-2}{2m} ds.$$

Therefore,

$$S^\star \mathrm{Ric}_0 = -\frac{m-2}{m} ds,$$

and comparison with $-S^\star S\theta = S^\star \mathrm{Ric}_0$ yields

$$S^\star S\theta = \frac{m-2}{m} ds.$$

Taking the L^2-inner product with θ and using that $S^\star$ is the formal adjoint of S, we get

$$\langle S^\star S\theta, \theta\rangle_{L^2} = \langle S\theta, S\theta\rangle_{L^2}.$$

On the other hand,

$$\langle ds, \theta\rangle_{L^2} = \int_M g(\nabla s, X) \, dv_g = \int_M \mathcal{L}_X s \, dv_g.$$

Hence the stated integral identity follows. If either hypothesis in the last assertion holds, the right-hand side vanishes, so $\|S\theta\|^2_{L^2} = 0$. Thus, $S\theta = 0$, and the remaining conclusions follow from (1.2) and $\mathrm{Ric}_0 = -S\theta$. □

The following theorem constitutes the main rigidity result of the paper.

Theorem 2.1. *Let (M, g, V, λ) be an m-dimensional $(m \geqslant 3)$ compact Riemann soliton. If $S^\star \operatorname{Ric}_0 = 0$, then (M, g) is Einstein.*

P r o o f. Since $\operatorname{Ric}_0 = -S\theta$, we have $S^\star \operatorname{Ric}_0 = -S^\star S\theta$. Thus, the assumption $S^\star \operatorname{Ric}_0 = 0$ implies $S^\star S\theta = 0$. Taking the L^2-inner product with θ gives $\|S\theta\|^2_{L^2} = 0$, and hence $S\theta = 0$. It follows from $\operatorname{Ric}_0 = -S\theta$ that $\operatorname{Ric}_0 = 0$. Therefore (M, g) is an Einstein manifold. □

The next result is an immediate application of the integral identity established in Proposition 2.3.

Corollary 2.2. *Let (M, g, V, λ) be an m-dimensional $(m \geqslant 3)$ compact Riemann soliton. If either s is constant or*

$$\int_M \mathcal{L}_X s \, dv_g = 0,$$

then X is a conformal vector field and (M, g) is Einstein.

P r o o f. Proposition 2.3 gives

$$\|S\theta\|^2_{L^2} = \frac{m-2}{m} \int_M \mathcal{L}_X s \, dv_g.$$

Under either hypothesis the right-hand side vanishes; hence, $S\theta = 0$. Using $\operatorname{Ric}_0 = -S\theta$, we obtain $\operatorname{Ric}_0 = 0$, while (1.2) shows that X is conformal. Thus, (M, g) is Einstein. □

Combining Corollary 2.2 with Obata's theorem gives the following classification result.

Corollary 2.3. *Let (M, g, V, λ) be an m-dimensional $(m \geqslant 3)$ compact Riemann soliton satisfying one of the assumptions of Corollary 2.2. If the induced conformal vector field X is essential, then (M, g) is globally isometric, up to homothetic normalization, to the standard round sphere S^m.*

P r o o f. By Corollary 2.2, (M, g) is Einstein. Since X is essential, it is non-Killing with respect to g; hence Obata's theorem (see [14]) applies and yields the conclusion. □

Finally, in dimension two the situation becomes completely rigid.

Theorem 2.2. *Let (M, g, V, λ) be a two-dimensional orientable compact Riemann soliton with constant scalar curvature s. Then:*

(i) *if $s > 0$, then (M, g) is globally isometric to a round two-sphere of constant Gaussian curvature $K = s/2$; equivalently, after the corresponding homothetic normalization, it is isometric to the standard unit sphere S^2;*

(ii) *if $s < 0$, then the Ricci almost soliton associated with (M, g, V, λ) by contraction is trivial; in fact, this conclusion is independent of the sign of s.*

P r o o f. On every two-dimensional Riemannian manifold, the scalar curvature s and the Gaussian curvature K are related by $s = 2K$. Hence, constant scalar curvature is equivalent to constant Gaussian curvature.

Assume first that $s > 0$. Then, $K = s/2 > 0$ is constant. By the Gauss–Bonnet theorem, $\chi(M) > 0$. Since M is compact and orientable, the classification of compact orientable surfaces implies that M is topologically a two-sphere. Because g has constant positive Gaussian curvature K, the Killing–Hopf theorem yields that (M, g) is globally isometric to the round two-sphere of curvature $K = s/2$. This proves (i).

For (ii), the direct contraction of the Riemann soliton equation in dimension two gives

$$\mathrm{Ric} = -(\lambda + \operatorname{div} V)g$$

and therefore

$$s = -2(\lambda + \operatorname{div} V).$$

Thus, the contracted equation is a Ricci almost soliton equation with vanishing potential vector field and soliton function $-(\lambda + \operatorname{div} V) = s/2$. Consequently, the associated Ricci almost soliton is trivial. The argument uses only the fact that the dimension is two and therefore does not depend on the sign of s; in particular, it applies when $s < 0$. This proves (ii). $\square$

Remark 2.1. If the orientability assumption in Theorem 2.2 is omitted, then in case (i) (M, g) is globally isometric either to a round sphere or to the real projective plane equipped with its standard constant-curvature metric, with Gaussian curvature $K = s/2$. Indeed, if M is non-orientable, its oriented double cover $\widetilde{M}$ is a compact orientable two-dimensional Riemann soliton with constant positive scalar curvature. By Theorem 2.2, $\widetilde{M}$ is isometric to the round sphere. The non-trivial deck transformation is therefore a fixed-point-free isometric involution of the round sphere, hence the antipodal map. Consequently,

$$M \cong S^2/\{\pm 1\} = \mathbb{R}P^2.$$

Acknowledgments

The authors express their deep gratitude to the memory of Professor Constantin Udrişte for his inspiring pioneering works on Riemann solitons, which laid the foundation for the present research. They are also highly grateful to the anonymous reviewer for their valuable comments and crucial remarks, which significantly improved the clarity and mathematical rigor of the manuscript.

Data availability. No datasets were generated or analyzed during the current study.

Funding. The authors declare that no financial funds, grants or other support were used in the preparation of this manuscript.

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Received 09.06.2026
Accepted 13.09.2026

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Citation: S. E. Stepanov, I. I. Tsyganok. Rigidity of compact Riemann solitons via the Ahlfors Laplacian, *Vestnik Udmurtskogo Universiteta. Matematika. Mekhanika. Komp'yuternye Nauki*, 2026, vol. 36, issue 3, pp. 486–495.

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Жесткость компактных солитонов Римана с помощью лапласиана Альфорса

Ключевые слова: солитон Римана, почти солитон Риччи, лапласиан Альфорса, оператор Коши–Альфорса, жесткость, многообразие Эйнштейна.

УДК 514.76

DOI: 10.35634/vm260309

Солитон Римана является жестким обобщением классического солитона Риччи, в определяющем уравнении которого участвует полный тензор кривизны Римана. В работе устанавливается связь между компактными солитонами Римана и операторным подходом, основанным на лапласиане Альфорса. С помощью свертки уравнения солитона Римана при размерности $m \geqslant 3$ строится ассоциированный почти солитон Риччи с нормированным потенциальным векторным полем. Получена глобальная формула, выражающая константу солитона через среднюю скалярную кривизну компактного многообразия. С помощью оператора Коши–Альфорса и формально сопряженного к нему оператора получены условия жесткости, при которых бесследовая часть тензора Риччи обращается в нуль и метрика является эйнштейновой. На основе теоремы Обаты сформулирован сферический результат жесткости. Отдельно рассмотрен двумерный случай: при положительной постоянной скалярной кривизне ориентируемый компактный солитон Римана является круглой сферой с соответствующей постоянной гауссовой кривизной, а ассоциированный почти солитон Риччи, возникающий при свертке, тривиален.

Финансирование. Авторы заявляют, что при подготовке настоящей статьи финансовая поддержка, гранты или иные формы финансирования не использовались.

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Поступила в редакцию 09.06.2026
Принята к публикации 13.09.2026

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Цитирование: С. Е. Степанов, И. И. Цыганок. Жесткость компактных солитонов Римана с помощью лапласиана Альфорса // Вестник Удмуртского университета. Математика. Механика. Компьютерные науки. 2026. Т. 36. Вып. 3. С. 486–495.