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ON CONSTRUCTING A STABILITY REGION FOR A LINEAR DISCRETE-TIME AUTONOMOUS EQUATION OF THE FOURTH ORDER

For an autonomous fourth-order linear difference equation, the stability domain S_4 in four-dimensional space is investigated, the parameters of which are the equation coefficients. Sections of the domain S_4 by hyperplanes of four-dimensional space are studied and constructed in three-dimensional space for a fixed value of the fourth parameter. It is shown that these sections are diffeomorphic to the stability domain S_3 of the third-order linear difference equation.

Keywords: linear difference equation, Schur stability, stability region.

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§ 1. Introduction

Consider an autonomous linear difference equation

$$x(n+k) + p_1 x(n+k-1) + p_2 x(n+k-2) + \ldots + p_k x(n) = 0, \quad n \in \mathbb{Z}, \tag{1}$$

where $x \in \mathbb{R}$ is a state, $p_1, \ldots, p_k$ are real numbers and $k \geqslant 1$ is an integer. The problem of the asymptotic stability of equation (1) is one of the most important problems in the theory of dynamic systems, from both theoretical and practical reasons. It has attracted the attention of many researchers. From (1), construct *the characteristic polynomial*

$$p(\lambda) = \lambda^k + p_1 \lambda^{k-1} + \cdots + p_k. \tag{2}$$

By λ_i, $i = \overline{1,k}$, denote the roots of (2); they are called *eigenvalues* of the equation (1). It is well known (see, e.g., [1, Section 5.1]) that the zero solution of (1) is asymptotically stable (or, equivalently, equation (1) is asymptotically stable) if and only if

$$\forall\, i = \overline{1,k} \quad |\lambda_i| < 1. \tag{3}$$

In this case, the polynomial (2) is called *Schur stable*. There are various criteria for checking the condition (3), for example, the Schur–Cohn criterion [1, Section 5.1], [2, Sections 3.4, 3.5], the Jury criterion [2, Sections 3.9, 3.10], the criterion using reduction to the Routh–Hurwitz (Liénard–Chipart) conditions [2, Section 3.3], and others.

The set $S_k = \{(p_1, \ldots, p_k)\} \subset \mathbb{R}^k$ of values of the coefficients $p_1, \ldots, p_k$ of equation (1), for which the equation is asymptotically stable, is called *the stability region* in the space $\mathbb{R}^k$. The problem of constructing a stability region has been studied in the works of many authors. Some sufficient conditions for asymptotic stability of equation (1) have been obtained in [3, 4]. In [5], necessary and sufficient conditions have been obtained for the asymptotic stability of the autonomous linear difference equation

$$x(n+1) + \alpha x(n) + \beta x(n-k+1) + \gamma x(n-k) = 0, \quad n \in \mathbb{Z},$$

where α, β, γ are real numbers and $k \geqslant 2$ is an integer, and a comparison of these stability conditions has been given with the corresponding results obtained by other authors in previous

works (see references in [5]). In [6], authors presented necessary and sufficient for the asymptotic stability of a three-term difference equation

$$x(n+2)+\alpha y(n)+\beta x(n-k)=0, \quad n\in\mathbb{Z}.$$

A certain type of necessary and sufficient conditions has been presented in [7] for the asymptotic stability of the linear difference equation

$$x(n)=\alpha x(n-m)+\beta x(n-k), \quad n\in\mathbb{Z},$$

where α, β are real scalars and $k>m>0$ are integers. A survey of the results on stability conditions for equation (1) was presented in [8]. Some sufficient conditions for the asymptotic stability of linear autonomous difference systems, including those with delays, have been obtained in the papers [9–12]. The issues of constructing stability regions of difference and differential equations, including equations with delays, were studied, in particular, in the papers [13–17]. The papers [18–21] are devoted to the study and construction of stability regions for linear difference equations with complex coefficients.

In this paper, we consider equation (1) of the fourth order. For this equation, we construct three-dimensional sections of the stability region S_4 with respect to one of the coefficient parameter of the equation.

§ 2. Preliminary results: $k<4$

($\mathbf{k=1}$) Let $k=1$. Then, clearly, the stability region $S_1\subset\mathbb{R}$ is equal to $(-1,1)$.

($\mathbf{k=2}$) Let $k=2$. Let us consider a second-order linear difference equation

$$x(n+2)+ax(n+1)+bx(n)=0, \quad n\in\mathbb{Z},$$

where $a,b\in\mathbb{R}$ are constant. It is well-known (see, e.g., [2, (3.46)–(3.48)]) that the stability region $S_2=\{(a,b)\}\subset\mathbb{R}^2$ is defined by the following system of inequalities:

$$\begin{cases} 1+a+b>0,\\ 1-a+b>0,\\ 1-b>0. \end{cases}$$

The stability region S_2 is a triangle (without boundaries) with vertices $N_1(-2,1)$, $N_2(2,1)$ and $N_3(0,-1)$; see Fig. 1.

($\mathbf{k=3}$) Let $k=3$. Let us consider a third-order linear difference equation

$$x(n+3)+ax(n+2)+bx(n+1)+cx(n)=0, \quad n\in\mathbb{Z}, \tag{4}$$

where $a,b,c\in\mathbb{R}$ are constant. The necessary condition for the Schur stability of the characteristic polynomial $p(\lambda)=\lambda^3+a\lambda^2+b\lambda+c$ is the following (see, e.g., [22]):

$$-3<a<3, \quad -3<b<3, \quad -1<c<1. \tag{5}$$

It is well-known (see, e.g., [2, (3.49)–(3.52)], [23]) that the stability region $S_3=\{(a,b,c)\}\subset\mathbb{R}^3$ is defined by the following system of inequalities:

$$\begin{cases} 1+a+b+c>0, & (6)\\ 1-a+b-c>0, & (7)\\ 1-b+ac-c^2>0, & (8)\\ 1+c>0, & (9)\\ 1-c>0. & (10) \end{cases}$$

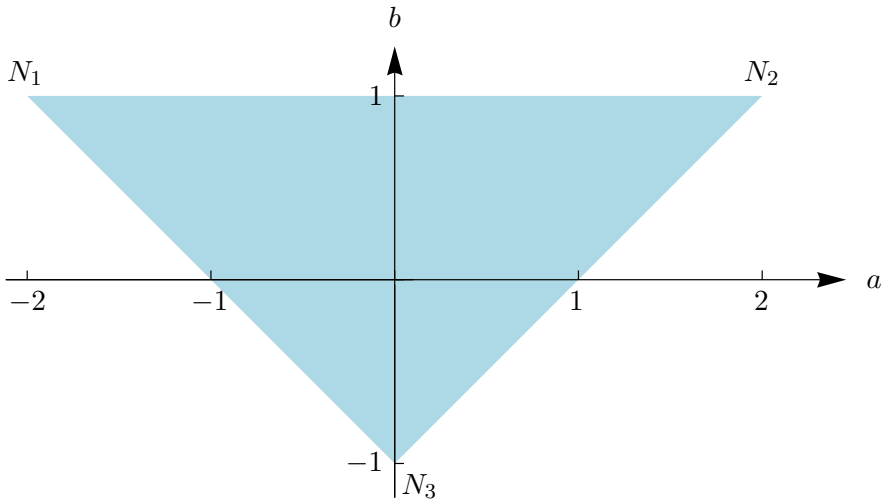

Fig. 1. The stability region S_2

From (6), (7), and (8), it follows, respectively, that

$$b > -1 - a - c, \tag{11}$$
$$b > -1 + a + c, \tag{12}$$
$$1 + ac - c^2 > b. \tag{13}$$

From (11), (13) and (12), (13), it follows that

$$1 + ac - c^2 > -1 - a - c,$$
$$1 + ac - c^2 > -1 + a + c,$$

or, equivalently,

$$2 + ac + a + c - c^2 > 0, \tag{14}$$
$$2 + ac - a - c - c^2 > 0. \tag{15}$$

Consider the quadratic equation corresponding to inequality (14):

$$c^2 - (a + 1)c - (a + 2) = 0. \tag{16}$$

Equation (16) has the solution:

$$c_1 = -1, \quad c_2 = a + 2. \tag{17}$$

Due to the first inequality of (5), we have $c_1 < c_2$. Hence, the solution of inequality (14) is:

$$c \in (-1, a + 2). \tag{18}$$

Consider the quadratic equation corresponding to inequality (15):

$$c^2 - (a - 1)c + (a - 2) = 0. \tag{19}$$

Equation (19) has the solution:

$$c_3 = a - 2, \quad c_4 = 1. \tag{20}$$

Due to the first inequality of (5), we have $c_3 < c_4$. Hence, the solution of inequality (15) is:

$$c \in (a - 2, 1). \tag{21}$$

From (18), the inequality (9) follows; from (21), the inequality (10) follows. Thus, inequalities (9) and (10) are a consequence of the system of inequalities (6), (7), and (8). So, system (6)–(10) is reduced to a system of inequalities:

$$\begin{cases} b > -a - c - 1, \\ b > a + c - 1, \\ b < 1 + ac - c^2. \end{cases}$$

The region S_3 is a bounded set. The convex hull of S_3 is a tetrahedron Θ with vertices $V_1(-1,-1,1)$, $V_2(3,3,1)$, $V_3(1,-1,-1)$, $V_4(-3,3,-1)$ in the space of variables $\{(a,b,c) \in \mathbb{R}^3\}$ (see [24, Example 2]). The region S_3 is bounded by the following hypersurfaces:

$$\gamma_1 : \quad b = -a - c - 1, \tag{22}$$
$$\gamma_2 : \quad b = a + c - 1, \tag{23}$$
$$\gamma_3 : \quad b = 1 + ac - c^2. \tag{24}$$

The surfaces γ_1 and γ_2 are planes. The surface γ_3 is a hyperbolic paraboloid. Let us find curves of intersection of surfaces (22), (23), and (24).

Let

$$l_{12} := \gamma_1 \cap \gamma_2 : \quad \begin{cases} b = -a - c - 1, \\ b = a + c - 1. \end{cases}$$

We obtain the line

$$l_{12} : \begin{cases} b = -1, \\ a + c = 0. \end{cases} \tag{25}$$

Let

$$l_{13} := \gamma_1 \cap \gamma_3 : \quad \begin{cases} b = -a - c - 1, \\ b = 1 + ac - c^2. \end{cases} \tag{26}$$

From system (26), the quadratic equation (16) follows. Its solution is (17). From this, taking into account (26), we obtain that the curve l_{13} consists of two lines: $l_{13} = l_{131} \cup l_{132}$,

$$l_{131} : \begin{cases} c = -1, \\ b = -a, \end{cases} \qquad l_{132} : \begin{cases} c = a + 2, \\ b = -2a - 3. \end{cases}$$

Let

$$l_{23} := \gamma_2 \cap \gamma_3 : \quad \begin{cases} b = a + c - 1, \\ b = 1 + ac - c^2. \end{cases} \tag{27}$$

From system (27), the quadratic equation (19) follows. Its solution is (20). From this, taking into account (27), we obtain that the curve l_{23} consists of two lines: $l_{23} = l_{231} \cup l_{232}$,

$$l_{231} : \begin{cases} c = 1, \\ b = a, \end{cases} \qquad l_{232} : \begin{cases} c = a - 2, \\ b = 2a - 3. \end{cases}$$

The projections of the edge curves of the region S_3 onto the plane Oac form the following line segments; see Fig. 2.

The point $A_1(-1,1)$ is a projection pr of the point $V_1(-1,-1,1)$ onto the plane Oac, i. e., $A_1(-1,1) = \mathrm{pr}\, V_1(-1,-1,1)$; next, $A_2(3,1) = \mathrm{pr}\, V_2(3,3,1)$, $A_3(1,-1) = \mathrm{pr}\, V_3(1,-1,-1)$; $A_4(-3,-1) = \mathrm{pr}\, V_4(-3,3,-1)$. The line segment A_1A_3 is a projection onto the plane Oac of

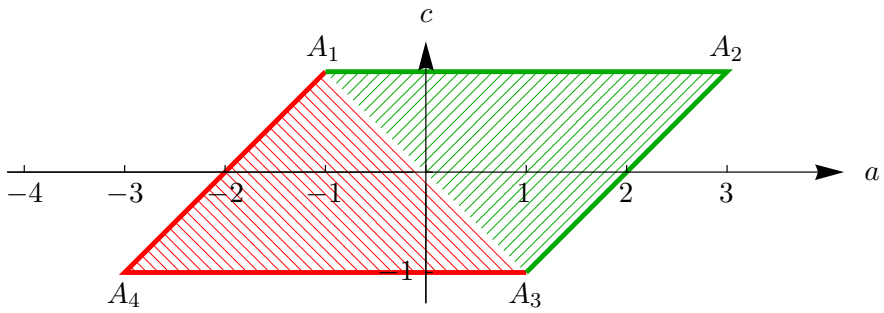

Fig. 2. The projections of the boundary edge curves of the region S_3 onto the plane Oac

the segment of the line l_{12}, which is the boundary edge for the region S_3 (let us write it like: $A_1A_3 = \mathrm{pr}\,(l_{12})$); next, $A_1A_2 = \mathrm{pr}\,(l_{231})$, $A_2A_3 = \mathrm{pr}\,(l_{232})$, $A_3A_4 = \mathrm{pr}\,(l_{131})$, $A_4A_1 = \mathrm{pr}\,(l_{132})$.

Let us place the coordinate b vertically. The region S_3 is shown in Fig. 3 (see also, e.g., [24–26]). If $(a, c) \in A_1A_2A_3$, then the region S_3 is bounded from below by the plane γ_2 (green plane in Fig. 3) and bounded from above by the surface γ_3. If $(a, c) \in A_3A_4A_1$, then the region S_3 is bounded from below by the plane γ_1 (red plane in Fig. 3) and bounded from above by the surface γ_3.

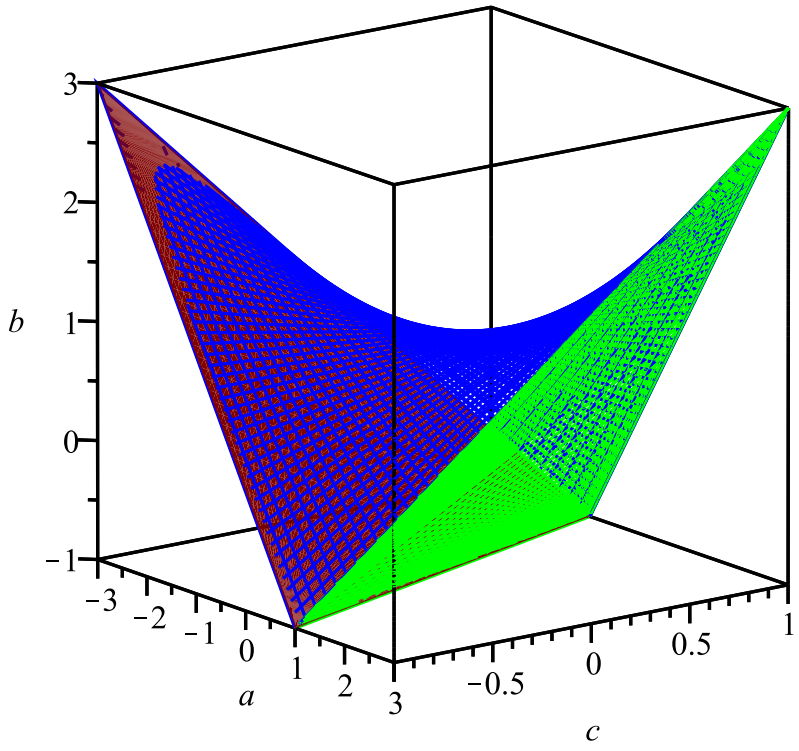

Fig. 3. The stability region S_3

§ 3. Main result

($\mathbf{k = 4}$) Let $k = 4$. Let us consider a fourth-order linear difference equation

$$x(n+4) + ax(n+3) + bx(n+2) + cx(n+1) + dx(n) = 0, \quad n \in \mathbb{Z}, \tag{28}$$

where $a, b, c, d \in \mathbb{R}$ are constant. The necessary condition for the Schur stability of the characteristic polynomial $p(\lambda) = \lambda^4 + a\lambda^3 + b\lambda^2 + c\lambda + d$ is the following (see [22]):

$$-4 < a < 4, \quad -6 < b < 6, \quad -4 < c < 4, \quad -1 < d < 1. \tag{29}$$

Denote by $S_4 = \{(a, b, c, d)\} \subset \mathbb{R}^4$ the stability region.

Consider the polynomials

$$\begin{aligned}
&(z+1)^4 = z^4 + 4z^3 + 6z^2 + 4z + 1,\\
&(z+1)^3(z-1) = z^4 + 2z^3 - 2z - 1,\\
&(z+1)^2(z-1)^2 = z^4 - 2z^2 + 1,\\
&(z+1)(z-1)^3 = z^4 - 2z^3 + 2z - 1,\\
&(z-1)^4 = z^4 - 4z^3 + 6z^2 - 4z + 1.
\end{aligned} \tag{30}$$

Let us consider points in four-dimensional space $\{(a, b, c, d) \in \mathbb{R}^4\}$ whose coordinates are equal to the coefficients of the polynomials (30). We get

$$Z_1(4, 6, 4, 1), \quad Z_2(2, 0, -2, -1), \quad Z_3(0, -2, 0, 1), \quad Z_4(-2, 0, 2, -1), \quad Z_5(-4, 6, -4, 1).$$

According to [24, Theorem 1], the convex hull of the set S_4 is a polyhedron in four-dimensional space with vertices at the points Z_1, Z_2, Z_3, Z_4, Z_5.

According to [2, (3.53)–(3.57)], the stability region S_4 is determined by the system of inequalities

$$\begin{cases}
1 + a + b + c + d > 0, & (31)\\
1 - a + b - c + d > 0, & (32)\\
-a^2 d + acd - bd^2 + d^3 + ac + 2bd - c^2 - d^2 - b - d + 1 > 0, & (33)\\
d^2 - 1 + ad - c < 0, & (34)\\
d^2 - 1 - ad + c < 0. & (35)
\end{cases}$$

The region S_4 is a set in $\mathbb{R}^4$. Due to (29), the region S_4 is contained in the four-dimensional strip $-1 < d < 1$ with respect to the fourth parameter. We will construct a section $S_4(d_0)$ of the set S_4 by the hyperplane $H(d_0) = \{d = d_0\} \subset \mathbb{R}^4$, that is, $S_4(d_0) := S_4 \cap H(d_0)$. Here the parameter d_0 runs through the set $(-1, 1)$. Then, the set $S_4(d_0) = \{(a, b, c)\}$ is contained in the three-dimensional space $\mathbb{R}^3$.

Let $d_0 \in (-1, 1)$ be fixed. Then, due to (31)–(35), the set $S_4(d_0)$ is defined by inequalities

$$\begin{cases}
b > f_1(a, c) := -(a + c) - (1 + d_0), & (36)\\
b > f_2(a, c) := (a + c) - (1 + d_0), & (37)\\
b < f_3(a, c) := (a - c)\dfrac{(c - ad_0)}{(1 - d_0)^2} + (1 + d_0), & (38)\\
c > h_1(a) := ad_0 + d_0^2 - 1, & (39)\\
c < h_2(a) := ad_0 - d_0^2 + 1. & (40)
\end{cases}$$

The region $S_4(d_0)$ is a region in the three-dimensional space $Oabc$, which is bounded by hypersurfaces

$$\delta_1: \quad b = -(a+c) - (1+d_0), \tag{41}$$
$$\delta_2: \quad b = (a+c) - (1+d_0), \tag{42}$$
$$\delta_3: \quad b = (a-c)\frac{(c-ad_0)}{(1-d_0)^2} + (1+d_0), \tag{43}$$

as (a, c) run through the set P on the plane Oac, which is bounded by the curves

$$\varphi_1: \quad c = ad_0 + d_0^2 - 1, \tag{44}$$
$$\varphi_2: \quad c = ad_0 - d_0^2 + 1. \tag{45}$$

The set $P \subset Oac$ is a strip enclosed between the lines (44) and (45) and containing $O(0,0)$ inside itself.

(a) Let $d_0 \in (0,1)$. Then, the lines φ_1 and φ_2 have an angle of slope $\alpha \in (0, \pi/4)$. See Fig. 4. The line φ_1 (the red line in Fig. 4) intersects the axis Oa at the point $Q_1\big((1-d_0^2)/d_0, 0\big)$ and the axis Oc at the point $Q_2(0, d_0^2 - 1)$. The line φ_2 (the green line in Fig. 4) intersects the axis Oa at the point $Q_3\big((d_0^2-1)/d_0, 0\big)$ and the axis Oc at the point $Q_4(0, 1-d_0^2)$.

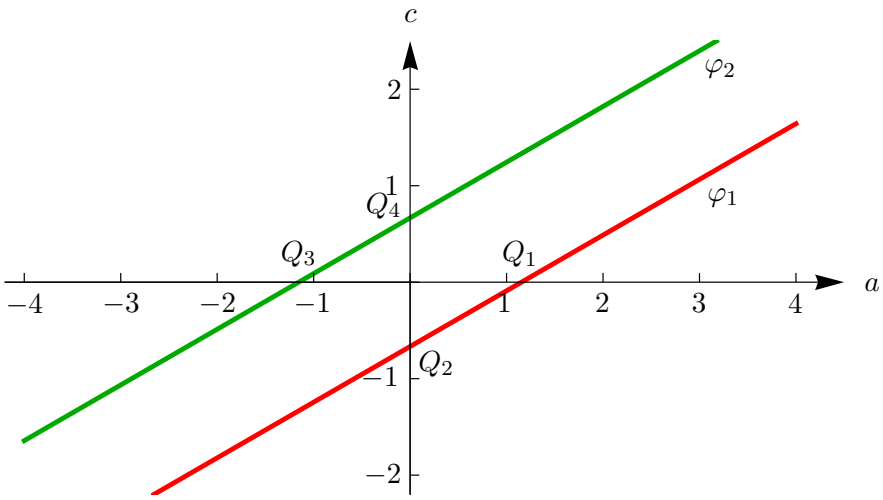

Fig. 4. The lines φ_1 and φ_2, $d_0 \in (0,1)$

(b) Let $d_0 = 0$. Then, the lines φ_1 and φ_2 have the form: $c = -1$ and $c = 1$. See Fig. 5. The line φ_1 (the red line in Fig. 5) intersects the axis Oc at the point $Q_5(0, -1)$. The line φ_2 (the green line in Fig. 5) intersects the axis Oc at the point $Q_6(0, 1)$.

(c) Let $d_0 \in (-1, 0)$. Then, the lines φ_1 and φ_2 have an angle of slope $\alpha \in (-\pi/4, 0)$. See Fig. 6. The line φ_1 (the red line in Fig. 6) intersects the axis Oa at the point $Q_7\big((1-d_0^2)/d_0, 0\big)$ and the axis Oc at the point $Q_8(0, d_0^2 - 1)$. The line φ_2 (the green line in Fig. 6) intersects the axis Oa at the point $Q_9\big((d_0^2-1)/d_0, 0\big)$ and the axis Oc at the point $Q_{10}(0, 1-d_0^2)$.

The surfaces δ_1 and δ_2 are planes. The surface δ_3 is a hyperbolic paraboloid. Let us find curves of intersection of surfaces (41), (42).

Let

$$m_{12} := \delta_1 \cap \delta_2: \quad \begin{cases} b = -(a+c) - (1+d_0), \\ b = (a+c) - (1+d_0). \end{cases}$$

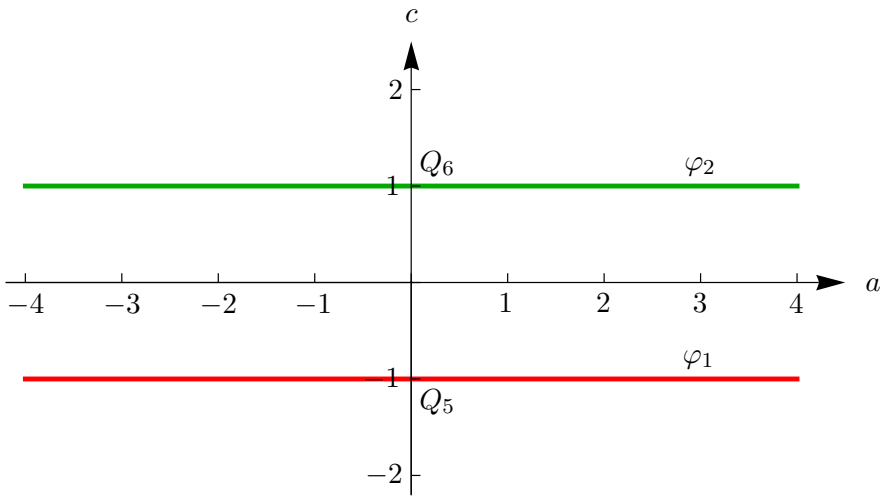

Fig. 5. The lines φ_1 and φ_2, $d_0 = 0$

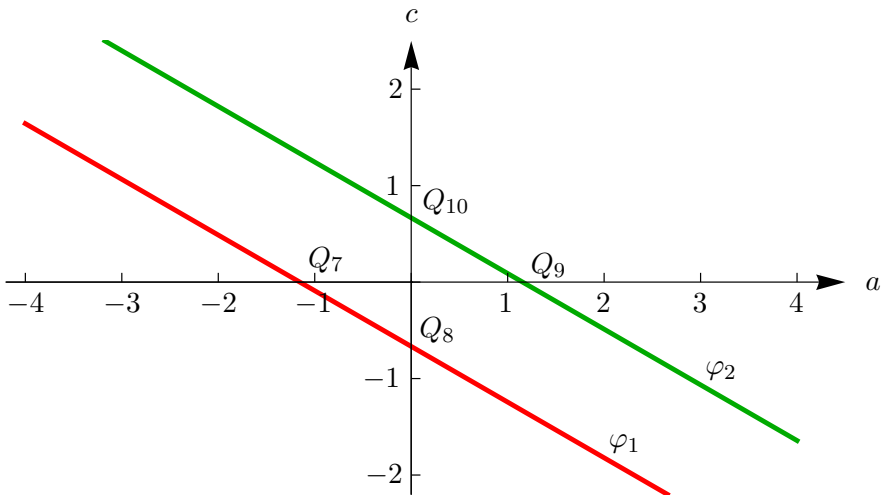

Fig. 6. The lines φ_1 and φ_2, $d_0 \in (-1, 0)$

We obtain the line

$$m_{12}: \begin{cases} b = -(1 + d_0), \\ a + c = 0. \end{cases} \tag{46}$$

If $d_0 = 0$, then (46) coincides with (25).

If $a + c < 0$, then $f_1(a, c) > f_2(a, c)$. Hence, due to (36), (37), and (38), the region $S_4(d_0)$ is bounded by δ_1 and δ_3.

If $a + c > 0$, then $f_2(a, c) > f_1(a, c)$. Hence, due to (36), (37), and (38), the region $S_4(d_0)$ is bounded by δ_2 and δ_3.

In the plane Oac, consider the line

$$\xi: \quad a + c = 0. \tag{47}$$

Let

$$B_1 := \varphi_2 \cap \xi: \quad \begin{cases} c = ad_0 - d_0^2 + 1, \\ c = -a. \end{cases}$$

Then, B_1 has the coordinates $(d_0 - 1, 1 - d_0)$.

Let

$$B_3 := \varphi_1 \cap \xi: \quad \begin{cases} c = ad_0 + d_0^2 - 1, \\ c = -a. \end{cases}$$

Then, B_3 has the coordinates $(1 - d_0, d_0 - 1)$. In particular, if $d_0 = 0$, then B_1 coincides with A_1 and B_3 coincides with A_3 (see Fig. 2).

The line (47) divides the strip P into two parts: in the left half-strip P_-, we have $a + c < 0$, and, in the right half-strip P_+, we have $a + c > 0$ (see Fig. 7, 8, 9 for $d_0 \in (0, 1)$, $d_0 = 0$, $d_0 \in (-1, 0)$, respectively). Let us consider these two cases separately.

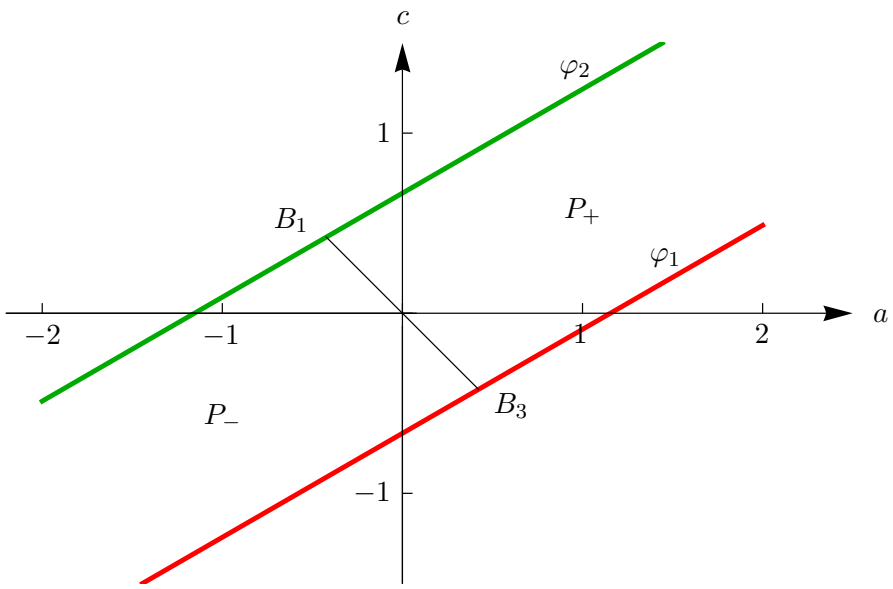

Fig. 7. The half-strips P_- and P_+, $d_0 \in (0, 1)$

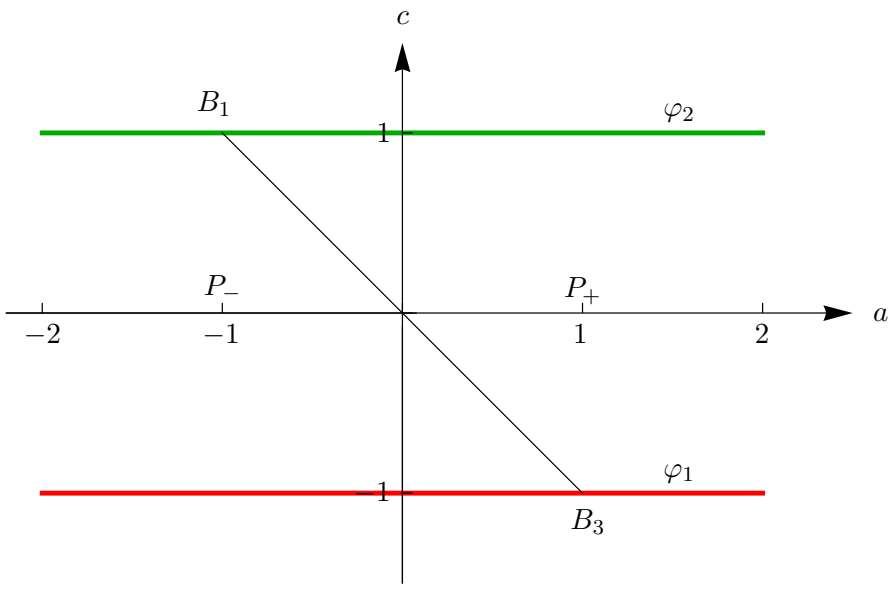

Fig. 8. The half-strips P_- and P_+, $d_0 = 0$

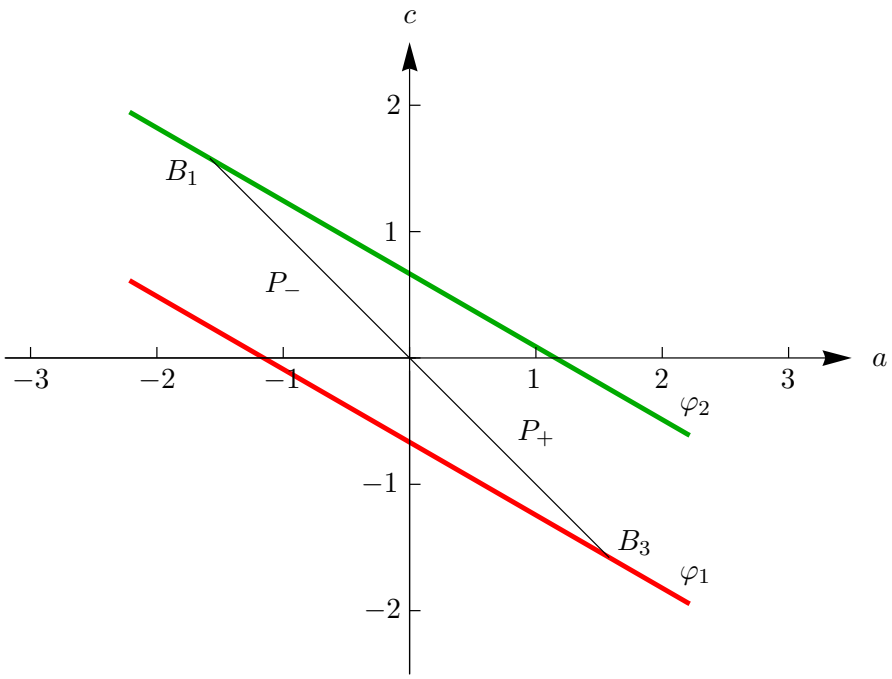

Fig. 9. The half-strips P_- and P_+, $d_0 \in (-1, 0)$

1. Let

$$a + c < 0, \quad h_1(a) < c < h_2(a). \tag{48}$$

The half-strip P_- is defined by (48). It is bounded by the half-line φ_1: $c = ad_0 + d_0^2 - 1$ $(a < 1 - d_0)$, by the half-line φ_2: $c = ad_0 - d_0^2 + 1$ $(a < d_0 - 1)$, and by the segment B_1B_3 of the line ξ. For $(a, c) \in P_-$, the region $S_4(d_0)$ is bounded by δ_1 and δ_3. Let us find curves of intersection of surfaces (41) and (43). Let

$$m_{13} := \delta_1 \cap \delta_3 : \quad \begin{cases} b = -(a + c) - (1 + d_0), \\ b = (a - c)\dfrac{(c - ad_0)}{(1 - d_0)^2} + (1 + d_0). \end{cases} \tag{49}$$

From system (49), we obtain a quadratic equation

$$(a - c)\frac{(c - ad_0)}{(1 - d_0)^2} + (1 + d_0) = -(a + c) - (1 + d_0). \tag{50}$$

After transforming (50), we obtain a quadratic equation

$$c^2 - c\big((d_0 - 1)^2 + a(d_0 + 1)\big) - a(d_0 - 1)^2 + a^2 d_0 - 2(d_0 - 1)^2(d_0 + 1) = 0. \tag{51}$$

Calculating the discriminant $\mathfrak{D}$ of the quadratic equation (51) we obtain that

$$\mathfrak{D} = (d_0^2 + ad_0 - a + 2d_0 - 3)^2.$$

The quadratic equation (51) has the solution

$$\left[\begin{array}{l} c = ad_0 + d_0^2 - 1, \\ c = a - 2d_0 + 2. \end{array}\right.$$

From this, taking into account (49), we obtain that the curve m_{13} consist of two lines: $m_{13} = m_{131} \cup m_{132}$,

$$m_{131}: \begin{cases} c = ad_0 + d_0^2 - 1, \\ b = -(a + d_0)(1 + d_0), \end{cases} \qquad m_{132}: \begin{cases} c = a + 2 - 2d_0, \\ b = -2a - 3 + d_0. \end{cases}$$

The projection of the line m_{131} onto the plane $Oac = \{b = 0\}$ coincides with the line

$$\varphi_1: \quad c = ad_0 + d_0^2 - 1.$$

The projection of the line m_{132} onto the plane $Oac = \{b = 0\}$ forms the line

$$\psi_1: \quad c = a + 2 - 2d_0. \tag{52}$$

Let us find the point of intersection of the lines ψ_1 and φ_1:

$$\begin{cases} c = a + 2 - 2d_0, \\ c = ad_0 + d_0^2 - 1. \end{cases}$$

We get the point $(-3 - d_0, -1 - 3d_0)$. Denote this point by B_4.

Let us find the point of intersection of the lines ψ_1 and φ_2:

$$\begin{cases} c = a + 2 - 2d_0, \\ c = ad_0 - d_0^2 + 1. \end{cases}$$

We get the point $\big(d_0 - 1, 1 - d_0\big)$. This point coincides with the point B_1.

The line (52) divides the half-strip P_- into two parts: in the left part P_-^1, we have $c > a + 2 - 2d_0$, and, in the right part P_-^2, we have $c < a + 2 - 2d_0$ (see Fig. 10, 11, 12 for $d_0 \in (0, 1)$, $d_0 = 0$, $d_0 \in (-1, 0)$, respectively).

It is easy to check the following properties:

(i) if $(a, c) \in P_-^2$, then $f_3(a, c) - f_1(a, c) > 0$; hence, $f_3(a, c) > f_1(a, c)$;

(ii) if $(a, c) \in \psi_1$, then $f_3(a, c) - f_1(a, c) = 0$; hence, $f_3(a, c) = f_1(a, c)$;

(iii) if $(a, c) \in P_-^1$, then $f_3(a, c) - f_1(a, c) < 0$; hence, $f_3(a, c) < f_1(a, c)$.

From this, it follows that, if $(a, c) \in P_-^2 = B_3B_4B_1$, then the region $S_4(d_0)$ is bounded from below by the plane δ_1 and bounded from above by the surface δ_3 (we suppose that the axis b is directed vertically); and if $(a, c) \in P_- \setminus P_-^2$, then system (36)–(40) has no solution, and for these values of (a, c) the set $S_4(d_0)$ is empty.

When the parameters (a, c) of the plane Oac are located at the vertices $B_1(d_0 - 1, 1 - d_0)$, $B_3(1 - d_0, d_0 - 1)$, $B_4(-3 - d_0, -1 - 3d_0)$ of the set P_-^2, the corresponding points $W_1, W_3, W_4 \in \in \{(a, b, c) \in \mathbb{R}^3\}$ of intersections of surfaces δ_1 and δ_3 have coordinates

$$W_1(d_0 - 1, -1 - d_0, 1 - d_0), \tag{53}$$
$$W_3(1 - d_0, -1 - d_0, d_0 - 1), \tag{54}$$
$$W_4(-3 - d_0, 3d_0 + 3, -1 - 3d_0).$$

2. Let

$$a + c > 0, \quad h_1(a) < c < h_2(a). \tag{55}$$

The half-strip P_+ is defined by (55). It is bounded by the half-line φ_1: $c = ad_0 + d_0^2 - 1$ $(a > 1 - d_0)$, by the half-line φ_2: $c = ad_0 - d_0^2 + 1$ $(a > d_0 - 1)$, and by the segment B_1B_3

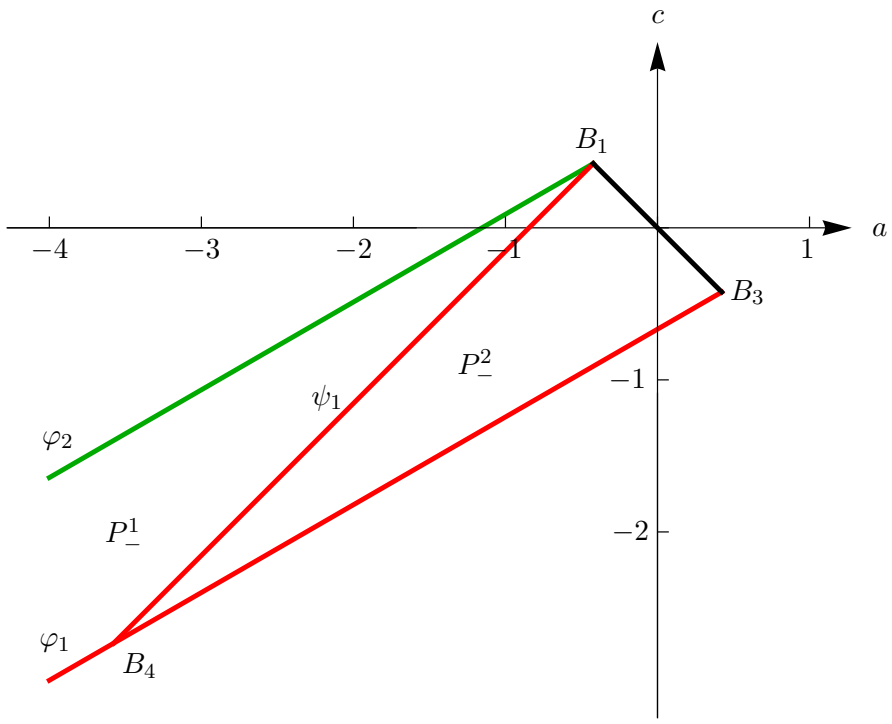

Fig. 10. The parts P_-^1 and P_-^2, $d_0 \in (0,1)$

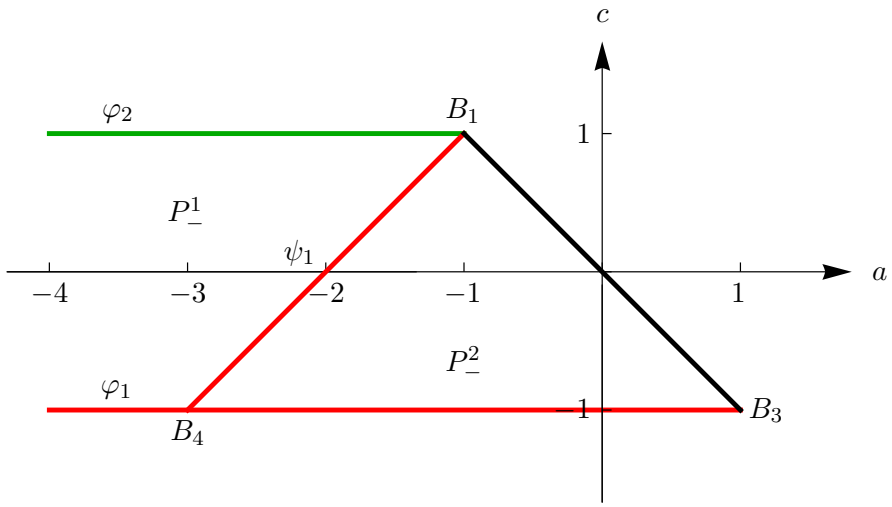

Fig. 11. The parts P_-^1 and P_-^2, $d_0 = 0$

of the line ξ. For $(a,c) \in P_+$, the region $S_4(d_0)$ is bounded by δ_2 and δ_3. Let us find curves of intersection of surfaces (42) and (43). Let

$$m_{23} := \delta_2 \cap \delta_3 : \quad \begin{cases} b = (a+c) - (1+d_0), \\ b = (a-c)\dfrac{(c-ad_0)}{(1-d_0)^2} + (1+d_0). \end{cases} \tag{56}$$

From system (56), we obtain a quadratic equation

$$(a-c)\frac{(c-ad_0)}{(1-d_0)^2} + (1+d_0) = (a+c) - (1+d_0). \tag{57}$$

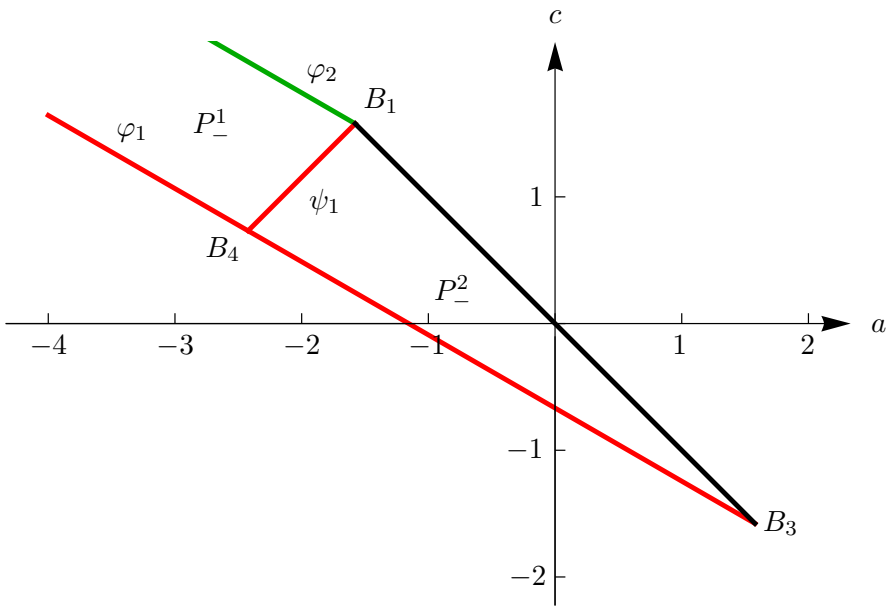

Fig. 12. The parts P_-^1 and P_-^2, $d_0 \in (-1, 0)$

After transforming (57), we obtain a quadratic equation

$$c^2 + c\big((d_0-1)^2 - a(d_0+1)\big) + a(d_0-1)^2 + a^2 d_0 - 2(d_0-1)^2(d_0+1) = 0. \tag{58}$$

Calculating the discriminant $\mathfrak{D}$ of the quadratic equation (58), we obtain that

$$\mathfrak{D} = (d_0^2 - ad_0 + a + 2d_0 - 3)^2.$$

The quadratic equation (58) has the solution

$$\left[\begin{array}{l} c = ad_0 - d_0^2 + 1, \\ c = a + 2d_0 - 2. \end{array}\right.$$

From this, taking into account (56), we obtain that the curve m_{23} consist of two lines: $m_{23} = m_{231} \cup m_{232}$.

$$m_{231}\colon \begin{cases} c = ad_0 - d_0^2 + 1, \\ b = (a - d_0)(1 + d_0), \end{cases} \qquad m_{232}\colon \begin{cases} c = a - 2 + 2d_0, \\ b = 2a - 3 + d_0. \end{cases}$$

The projection of the line m_{231} onto the plane $Oac = \{b = 0\}$ coincides with the line

$$\varphi_2\colon \quad c = ad_0 - d_0^2 + 1.$$

The projection of the line m_{232} onto the plane $Oac = \{b = 0\}$ forms the line

$$\psi_2\colon \quad c = a - 2 + 2d_0. \tag{59}$$

Let us find the point of intersection of the lines ψ_2 and φ_2:

$$\begin{cases} c = a - 2 + 2d_0, \\ c = ad_0 - d_0^2 + 1. \end{cases}$$

We get the point $(3 + d_0, 1 + 3d_0)$. Denote this point by B_2.

Let us find the point of intersection of the lines ψ_2 and φ_1:

$$\begin{cases} c = a - 2 + 2d_0, \\ c = ad_0 + d_0^2 - 1. \end{cases}$$

We get the point $\big(1 - d_0, d_0 - 1\big)$. This point coincides with the point B_3.

The line (59) divides the half-strip P_+ into two parts: in the left part P_+^1, we have $c > a - 2 + 2d_0$, and, in the right part P_+^2, we have $c < a - 2 + 2d_0$ (see Fig. 13, 14, 15 for $d_0 \in (0, 1)$, $d_0 = 0$, $d_0 \in (-1, 0)$, respectively).

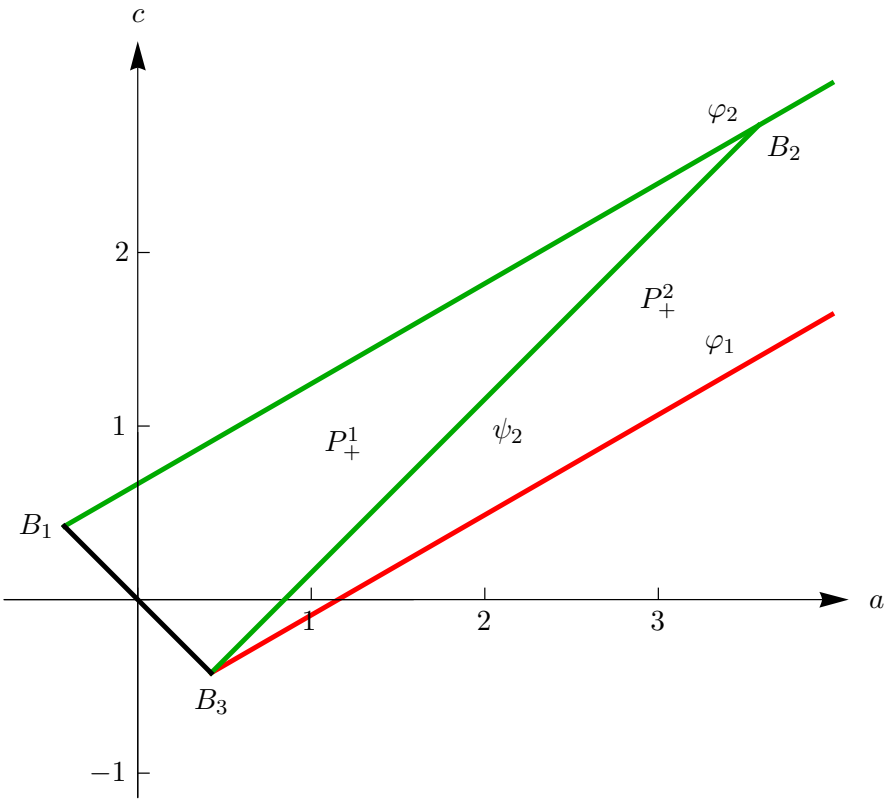

Fig. 13. The parts P_+^1 and P_+^2, $d_0 \in (0, 1)$

It is easy to check the following properties:

(i) if $(a, c) \in P_+^1$, then $f_3(a, c) - f_2(a, c) > 0$; hence, $f_3(a, c) > f_2(a, c)$;

(ii) if $(a, c) \in \psi_2$, then $f_3(a, c) - f_2(a, c) = 0$; hence, $f_3(a, c) = f_2(a, c)$;

(iii) if $(a, c) \in P_+^2$, then $f_3(a, c) - f_2(a, c) < 0$; hence, $f_3(a, c) < f_2(a, c)$.

From this, it follows that, if $(a, c) \in P_+^1 = B_1B_2B_3$, then the region $S_4(d_0)$ is bounded from below by the plane δ_2 and bounded from above by the surface δ_3 (we suppose that the axis b is directed vertically); and if $(a, c) \in P_+ \setminus P_+^1$, then system (36)–(40) has no solution, and for these values of (a, c) the set $S_4(d_0)$ is empty.

When the parameters (a, c) of the plane Oac are located at the vertices $B_1(d_0 - 1, 1 - d_0)$, $B_2(3 + d_0, 1 + 3d_0)$, $B_3(1 - d_0, d_0 - 1)$ of the set P_+^1, the corresponding points $W_1, W_2, W_3 \in \in \{(a, b, c) \in \mathbb{R}^3\}$ of intersections of surfaces δ_2 and δ_3 have coordinates (53), (54) and

$$W_2(3 + d_0, 3d_0 + 3, 1 + 3d_0).$$

The tetrahedron $W_1W_2W_3W_4$ forms the convex hull of the set $S_4(d_0)$. The projection of the set $S_4(d_0)$ onto the plane $Oac = \{b = 0\}$ forms the parallelogram $B_1B_2B_3B_4$, whose sides lie on

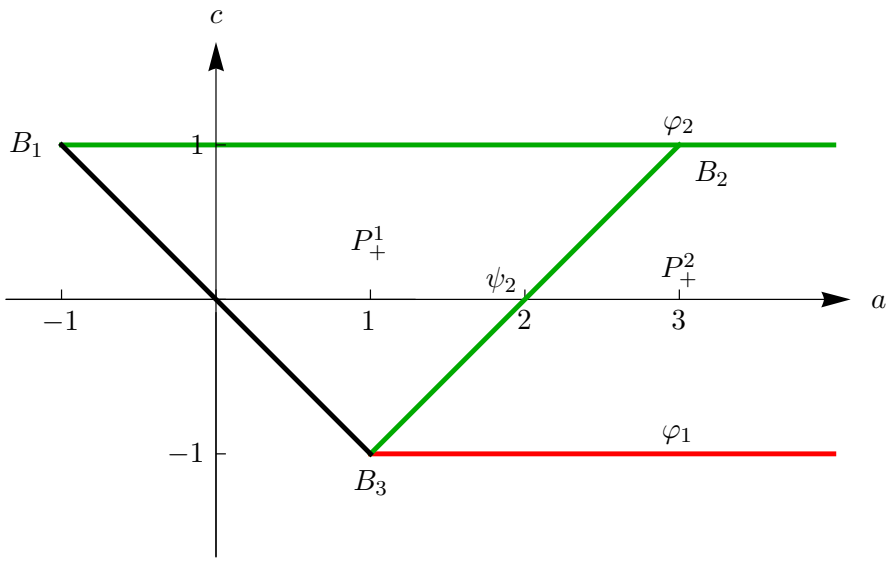

Fig. 14. The parts P_+^1 and P_+^2, $d_0 = 0$

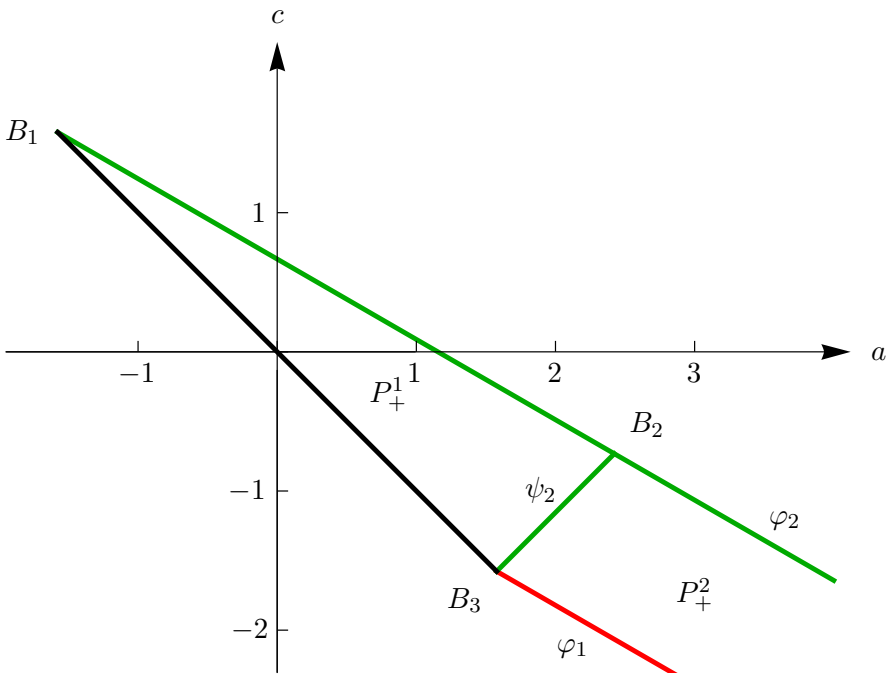

Fig. 15. The parts P_+^1 and P_+^2, $d_0 \in (-1, 0)$

the lines φ_1, φ_2, ψ_1, ψ_2 (see Fig. 16, 17, 18): $B_1B_2 \in \varphi_2$, $B_2B_3 \in \psi_2$, $B_3B_4 \in \varphi_1$, $B_4B_1 \in \psi_1$. The region $S_4(d_0)$ is drawn in Fig. 19, 20, 21 for $d_0 \in (0, 1)$, $d_0 = 0$, $d_0 \in (-1, 0)$, respectively.

So, we have the following theorem.

Theorem 1. *For an autonomous fourth-order linear difference equation* (28)*, the stability region* $S_4 \subset \mathbb{R}^4$ *is defined by the system of inequalities* (31)–(35)*. For the fixed* $d = d_0 \in (-1, 1)$*, the section* $S_4(d_0) = S_4 \cap H(d_0) \subset \mathbb{R}^3$ *of the stability domain* S_4 *by the hyperplane* $H(d_0) = \{d = d_0\} \subset \mathbb{R}^4$ *is a three-dimensional connected open set. This set has the following properties:*

(a) *the convex hull of the set* $S_4(d_0) = \{(a, b, c)\} \subset \mathbb{R}^3$ *is the tetrahedron* $W_1W_2W_3W_4$ *with*

the vertices

$$W_1(d_0-1,-1-d_0,1-d_0),\quad W_2(3+d_0,3d_0+3,1+3d_0),$$
$$W_3(1-d_0,-1-d_0,d_0-1),\quad W_4(-3-d_0,3d_0+3,-1-3d_0);$$

(b) *the projection of the region* $S_4(d_0)$ *onto the plane* $Oac=\{b=0\}$ *forms a parallelogram* $B_1B_2B_3B_4$ *with vertices*

$$B_1(d_0-1,1-d_0),\ B_2(3+d_0,1+3d_0),\ B_3(1-d_0,d_0-1),\ B_4(-3-d_0,-1-3d_0);$$

the sides B_3B_4, B_1B_2, B_4B_1, B_2B_3 *of this parallelogram lie, respectively, on lines*

$$\begin{aligned}\varphi_1:&\ c=ad_0+d_0^2-1,\\ \varphi_2:&\ c=ad_0-d_0^2+1,\\ \psi_1:&\ c=a+2-2d_0,\\ \psi_2:&\ c=a-2+2d_0;\end{aligned}$$

(c) *in the coordinate system* (a,c,b)*: if* $(a,c)\in B_1B_3B_4$*, then the set* $S_4(d_0)$ *is bounded from below by the plane*

$$\delta_1:\quad b=-(a+c)-(1+d_0)$$

and bounded from above by the hyperbolic paraboloid

$$\delta_3:\quad b=(a-c)\frac{(c-ad_0)}{(1-d_0)^2}+(1+d_0);$$

if $(a,c)\in B_1B_2B_3$*, then the set* $S_4(d_0)$ *is bounded from below by the plane*

$$\delta_2:\quad b=(a+c)-(1+d_0),$$

and bounded from above by the hyperbolic paraboloid

$$\delta_3:\quad b=(a-c)\frac{(c-ad_0)}{(1-d_0)^2}+(1+d_0);$$

(d) *for every* $d_0\in(-1,1)$*, the set* $S_4(d_0)$ *is diffeomorfic to the stability region* $S_3\subset\mathbb{R}^3$ *for equation* (4)*;*

(e) *for* $d_0=0$*, the set* $S_4(d_0)$ *coincides with* S_3*.*

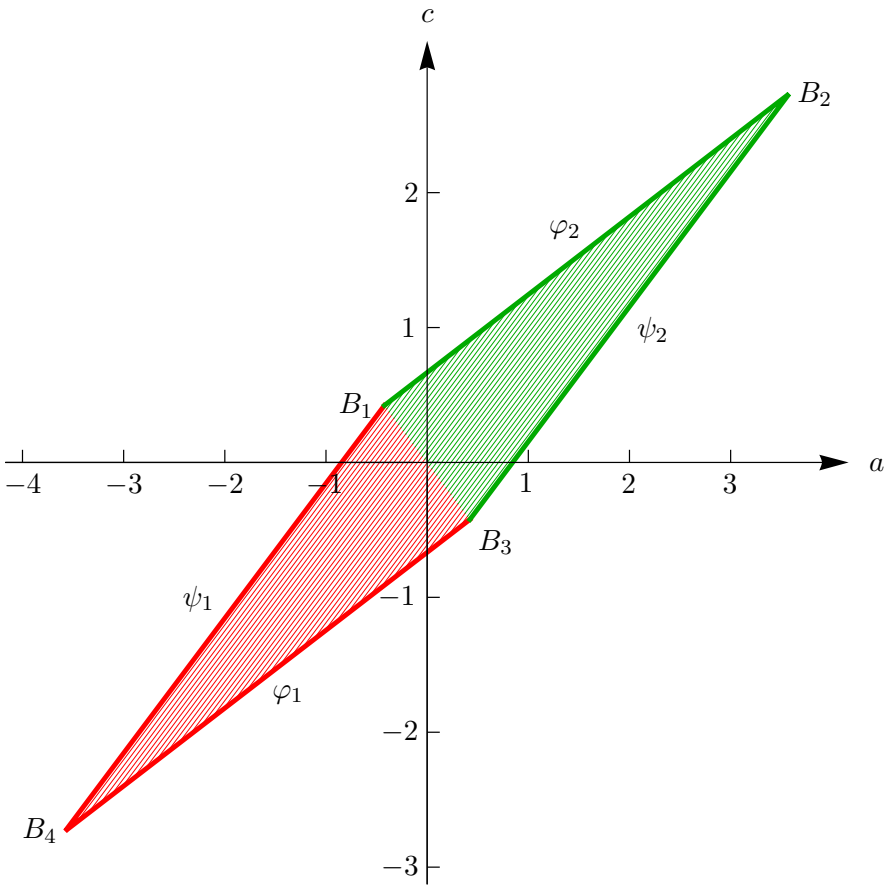

Fig. 16. The projection of the closure of $S_4(d_0)$ onto the plane Oac for $d_0 \in (0, 1)$

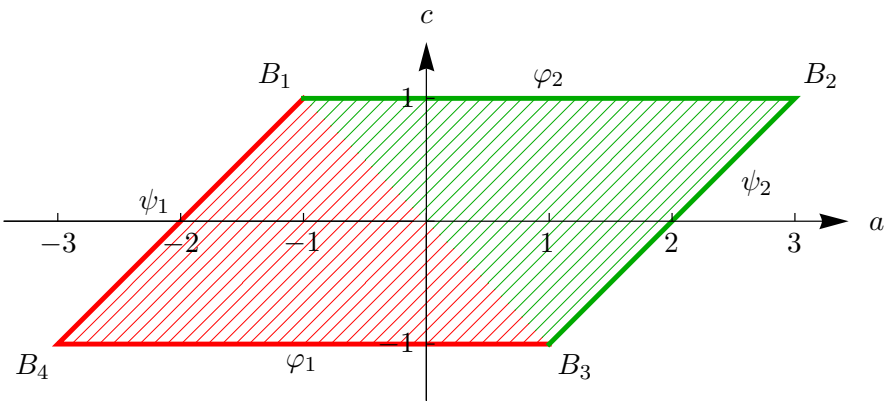

Fig. 17. The projection of the closure of $S_4(d_0)$ onto the plane Oac for $d_0 = 0$

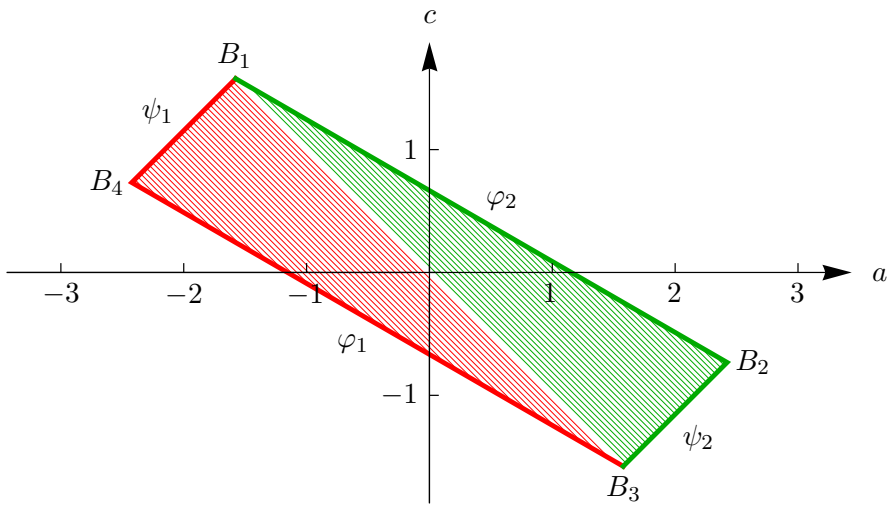

Fig. 18. The projection of the closure of $S_4(d_0)$ onto the plane Oac for $d_0 \in (-1, 0)$

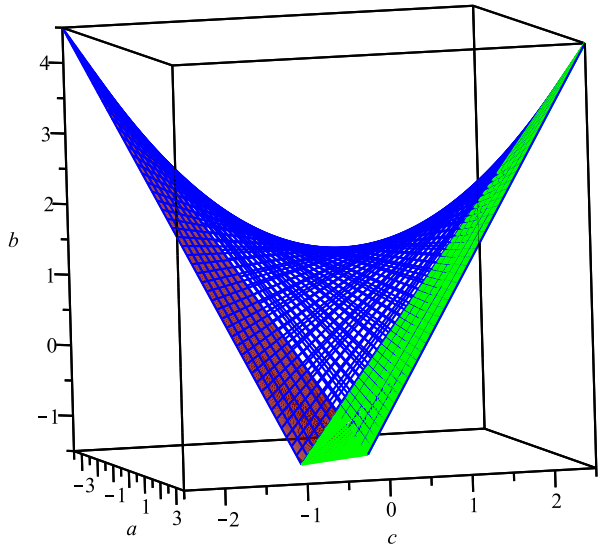

Fig. 19. The region $S_4(d_0)$ for $d_0 \in (0, 1)$

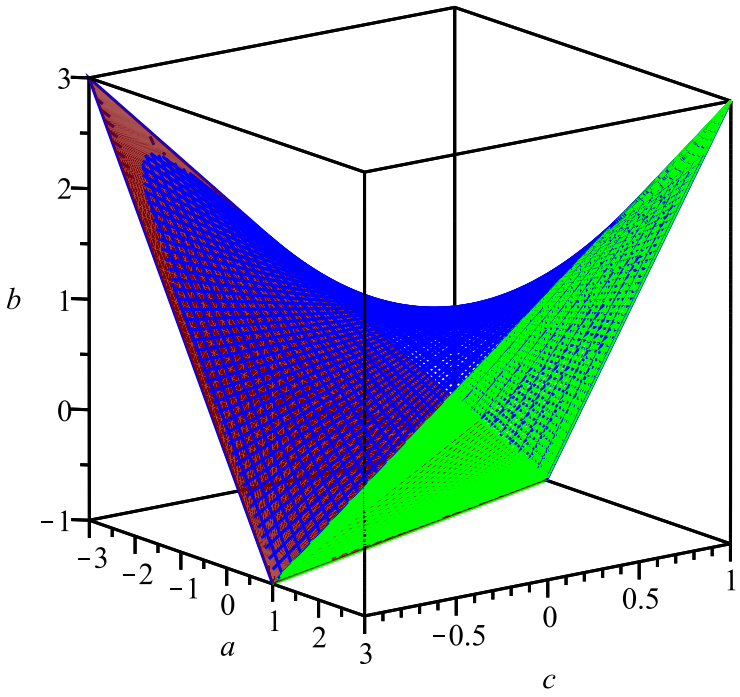

Fig. 20. The region $S_4(d_0)$ for $d_0 = 0$

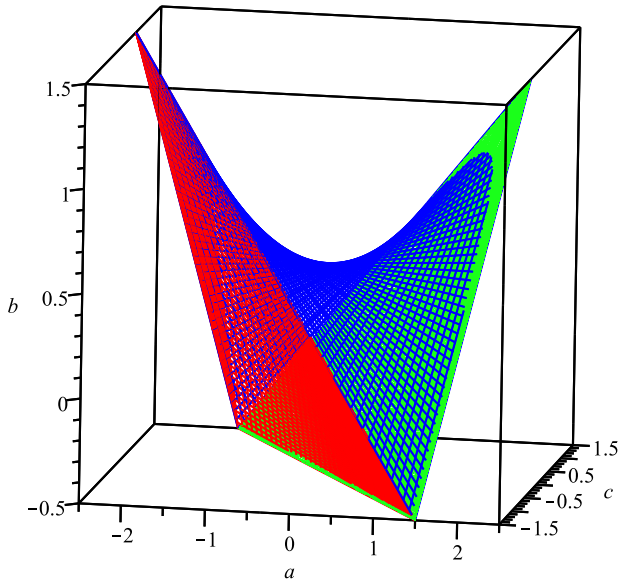

Fig. 21. The region $S_4(d_0)$ for $d_0 \in (-1, 0)$

§ 4. Conclusion

We have investigated the stability domain $S_4 \subset \mathbb{R}^4$ for the autonomous fourth order linear difference equation (28). We have constructed sections $S_4(d_0) = S_4 \cap H(d_0) \subset \mathbb{R}^3$ of the stability domain S_4 by the hyperplanes $H(d_0) = \{d = d_0\} \subset \mathbb{R}^4$ when the parameter d_0 varies from -1 to 1. It has been proven that the obtained sets are diffeomorphic to the stability domain $S_3 \subset \mathbb{R}^3$ for the autonomous third-order linear difference equation (4). It is of interest to study the sections of the set S_4 by hyperplanes with respect to other parameters of the equation (28). This may be a task for further research. It seems that the resulting sections will have a more complex structure and such diffeomorphism will not appear.

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К вопросу построения области устойчивости для линейного дискретного автономного уравнения четвертого порядка

Ключевые слова: линейное разностное уравнение, устойчивость по Шуру, область устойчивости.

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Для автономного линейного разностного уравнения четвертого порядка исследуется область устойчивости S_4 в четырехмерном пространстве, параметрами которого являются коэффициенты уравнения. Исследованы и построены в трехмерном пространстве сечения области S_4 гиперплоскостями четырехмерного пространства при фиксированном значении четвертого параметра. Показано, что эти сечения диффеоморфны области устойчивости S_3 линейного разностного уравнения третьего порядка.

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